Ex 11.3.1: a) By the formula

$$\sum_{v\in V} \deg(v) = 2\cdot|E|$$

$$3|V| = \sum_{v\in V} 3 = 2\cdot 9$$

So we get $|V| = 6$.

b) Again we have:

$$\sum_{v\in V} \deg(v) = 2.|E| = 30$$

Suppose the graph is k-regular then

$$k|V| = 30$$

So k and $|V|$ can be any divisor of 30.

c) The formula gives

$$\sum_{v\in V} \deg(v) = 20$$

$$2\cdot 4 + (|V|-2)\,3 = 20$$

$$3|V| - 6 = 12$$

So $|V| = 6$.

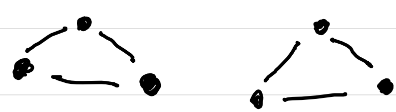
<u>Ex 11.3.2</u>: We have $\sum_{v\in V} \deg(v) = 34$

$$3|V| \le 34$$

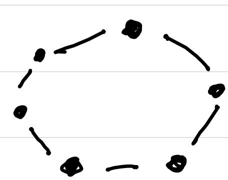
$$|V| \le 11$$

since $|V|$ is an integer.

<u>Ex. 11.3.4</u>: a)



This is one possibility.



This is the other possibility.

b) We get a disjoint union of a 3 cycle and a 4-cycle or a 7-cycle.

c) If we have one such graph G_1 we can check that it's compliment in K_r is 2-regular.
Also two graphs are isomorphic if and only if there complements are.

From question a) we know the answer is 2.

d) For the same reasons the answer is 2 again.

e) From a) and b) we get that a 2-regular graph is a disjoint union of cycles. If the graph has $|V|=n$ vertices the number of such possible graphs is the number of partitions of n where every part is at least 3. By the complement argument the number of $n-3$-regular graphs on n vertices is the same.

Ex 11.3.8: a) We have $|V| = 2^8$ and the graph is 8-regular. So we get

$$2^8 \cdot 8 = 2 \cdot |E|$$

so $|E| = 2^{10}$.

b) The maximum distance is 8 and is obtained by $0 \cdots 0$ $\cdot$
$1 \cdots 1$

c) Starting Q_2 we can get

$$(0,0) - (0,1) - (1,1) - (1,0)$$

with Q_1: $0 - 1$

If we are given a path for Q_n that goes through all vertices
$$v_1 - v_2 - \cdots v_{2^n}$$
we can produce a path for Q_{n+1} in the following way:

$$0v_1 - 0v_2 - \cdots - 0v_{2^n} - 1v_{2^n} - 1v_{2^n-1} - \cdots - 1v_1.$$

Since we have such a paths for Q_2 we get one for all Q_n.

Ex 11.3.10: We look for paths of the form

$$v_1 - v_2 - v_3$$

2^n choices $\quad$ n choices $\quad$ $n-1$ choices

So the total number is $\frac{2^n n(n-1)}{2}$

Ex 11.3.22: Euler trail $\iff$ 2 vertices of odd degree

vertex	deg
a	3
b	5
c	3
d	3

We can remove any edge, for example $c-d$ to get: $\deg(a) = 3$, $\deg(b) = 5$
$\deg(c) = \deg(d) = 2$

so an Euler trail by the theorem.

Ex 11.3.24: Every edge contributes to 1 in both in and out total degrees so:

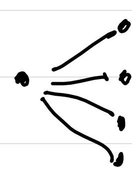
$$\sum_{v\in V} id(v) = \sum_{v\in V} od(v) = |E|.$$

Ex 11.4.2: By Kuratowski's theorem if we remove one edge of each graph we get a planar graph.

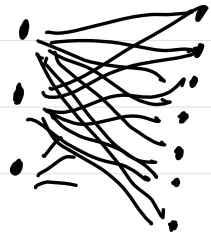
Ex 11.4.6: To produce $K_{1,3}$ in K_n

choose a first vertex v_1 (n choices) and then 3 vertices in the $n-1$ left $\binom{n-1}{3}$.

We get $n\binom{n-1}{3}$.

Ex 11.4.8: a)  $\rightsquigarrow$ 2

b)

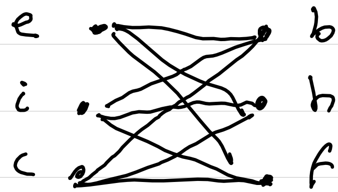
 $\rightsquigarrow 6$

c) d) For $K_{7,12}$ or $K_{m,n}$ we have

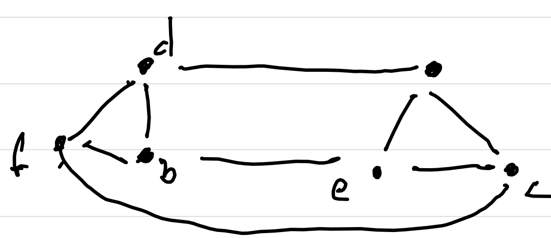
14 or $2m$ as longest path.

Ex 11.4.10: No, this is not possible since each movement swap sides in the graph and we have to end at the side we started with.

Ex 11.4.14: a) Non planar. Remove a, d, j, g



This is indeed $K_{3,3}$ so

b) 

Planar!

Ex 11.4.18: By Euler's relation:

$$v = 2 - r + e$$

From the assumption:

$$2e \leq 5r$$

so

$$v \geq 2 - r + \frac{5r}{2}$$

$$v \geq 2 + \frac{3r}{2}$$

We get $|V| \geq 82$.