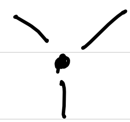
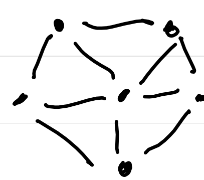
Ex 11.5.2: We can not have a vertice of degree 3:



In the first case the graph is a path.
In the second case the graph is a cycle.

Ex 11.5.6: For example on W_5:



A Hamilton cycle is determined by a choice of an outer edge that you don't go through so there are n choices for W_n.

Ex 11.5.9 : a) We start by choosing a vertex on one side ($2n$ choices). Then there are n vertices on the other side to choose, going back we have $n-1$ choices twice, then $n-2$ choices etc. We divide by 2 at the end.

$$\rightsquigarrow \frac{(n!)^2}{2} \quad \left(\text{divide by } n^2\right)$$

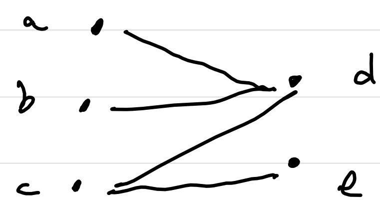
b) To get a path we remove one edge: n^2 choices:

$$\rightsquigarrow \text{(book)} \quad n^2 \frac{(n-1)!\, n!}{2} = \frac{n(n!)^2}{2}.$$

Ex 11.5.10 : a) For such a cycle we have to swap between V_1 and V_2 for every step so we must pass through both parts equally many times.

b) From a) since we can extend a Hamilton path in a cycle we must have $|V_1| = |V_2|$.
(example possible with $K_{2,2}$)

c)



<u>Ex 11.5-12</u>: We have already seen how to get such a path.
Start with such a path for Q_n we get one for Q_{n+1} ($v_1 - v_2 \cdots v_{2^n}$) by:

$$0v_1 - 0v_2 - \cdots 0v_{2^n} - 1v_{2^n} - \cdots - 1v_1$$

<u>Ex 11.5-18</u>: Make a graph G with $V = \{\text{people}\}$ $E = \{\text{relations}\}$. Then every vertex has degree at least 6.
So $\deg v \geq \frac{|V|}{2}$ for every $v \in V$ and

there is a Hamilton cycle.

__Ex. 11.6.2__: Make a graph G with $V = \{\text{committees}\}$ and $E = \{\text{committees with a common member}\}$.

The number we want is $\chi(G)$.

__Ex. 11.6.4__: Let's consider the 5-cycle:

then $\chi(G) = 3$.

__Ex. 11.6.6__: a) (i) In this case we get

$$\lambda(\lambda-1)^3 \text{ colorings}$$

(ii) In this case we get $\lambda(\lambda-1)(\lambda-2)^3$ colorings.

b) The chromatic polynomial

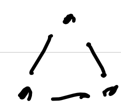
$$P(K_{2,3}, \lambda) = \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)^3,$$

and the chromatic number is 2.

c) The chromatic polynomial for $K_{2,n}$:

$$P(K_{2,n},\lambda) = \lambda(\lambda-1)^n + \lambda(\lambda-1)(\lambda-2)^n$$

and $\chi(K_{2,n}) = 2$.

Ex. 11.6.15: a) We have $C_3 =$ 

and $P(C_3,\lambda) = \lambda(\lambda-1)(\lambda-2)$.

b) If $n \geq 4$ from the result of the course by removing any edge we get:

$$P(C_n,\lambda) = P(P_{n-1},\lambda) - P(C_{n-1},\lambda)$$

c) The chromatic polynomial for the path graph:

$$P(P_{n-1},\lambda) = \lambda(\lambda-1)^{n-2}.$$

For $n \geq 4$:

d) $P(C_n, \lambda) = P(P_{n-1}, \lambda) - P(C_{n-1}, \lambda)$

$$P(C_n, \lambda) = \lambda(\lambda-1)^{n-2} - P(C_{n-1}, \lambda)$$

$$P(C_n, \lambda) = \lambda(\lambda-1)^{n-2} + (\lambda-1)^{n-1} - (\lambda-1)^{n-1} - P(C_{n-1}, \lambda)$$

$$P(C_n, \lambda) = (\lambda-1)(\lambda-1)^{n-2} + (\lambda-1)^{n-1} - P(C_{n-1}, \lambda)$$

$$P(C_n, \lambda) - (\lambda-1)^n = (\lambda-1)^{n-2} - P(C_{n-1}, \lambda)$$

For $n \geq 5$: replace $(\lambda-1)^{n-1} - P(C_{n-1}, \lambda)$ by the previous formula

$$P(C_n, \lambda) - (\lambda-1)^n = P(C_{n-2}, \lambda) - (\lambda-1)^{n-2}.$$

e) By induction:

$$P(C_{n+1}, \lambda) = \lambda(\lambda-1)^n - ((\lambda-1)^n + (-1)^n(\lambda-1))$$

$$= (\lambda-1)^{n+1} + (-1)^{n+1}(\lambda-1).$$

Ex 11.6.16: a) We have:

$$\chi(W_n) = \chi(C_n) + 1.$$

b) We have λ choices for the center and then we choose a proper coloring of the outer cycle with $\lambda - 1$ colors.

So: $P(W_n, \lambda) = \lambda P(C_n, \lambda - 1)$

$$= \lambda\left((\lambda-2)^n + (-1)^n(\lambda-2)\right).$$

c) i) there are $P(W_5, k)$ such colorings.

ii) $\chi(W_5) = 3$.

Ex 11.6.10: a) The graphs are not isomorphic since one has a vertice of degree 2 and not the other.

b) $P(G, \lambda) = \lambda(\lambda-1)^2(\lambda-2)^3$

c) The chromatic polynomial does not determine the isomorphism class of graphs.