Ex 12.1.2: For trees $|V_1| = |E_1| + 1$

so $|V_1| = 18$, $|V_2| = 36$, $|E_2| = 35$.

Ex 12.1.3 a) Since we have 7 components that are trees we have

$$|V_1| = |E_1| + 7$$

so $|V_1| = 47$.

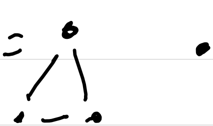
b) We have $|V_2| = 62$ and $|E_2| = 51$

so $|V_2| = |E_2| + 11$ which gives 11 components.

Ex 12.1.4: Same as previous exercise.

$$v = e + k.$$

Ex 12.1.5: These are paths.

Ex 12.1.7: $G =$  works.

Ex 12.1.8: a) We have on one side

$$\sum_{v} \deg(v) = 2|E|$$

$$4\times 2 + 3 + 2\times 4 + 5 + n = 2|E|$$
$$24 + n = 2|E|$$

and on the other side

$$|V| = |E| + 1$$
$$4 + 1 + 2 + 1 + n = |E| + 1$$
$$|E| = 7 + n$$

So $$24 + n = 2(7+n)$$
$$10 + n = 2n$$
and $n = 10$.

b) We have

- $\sum_{v} \deg v = v_1 + 2v_2 + 3v_3 + \dots + m v_m = 2|E|$
- $|E| + 1 = v_1 + v_2 + \dots + v_m$

From $2|E| = 2V_1 + 2V_2 + \dots + 2V_m - 2$

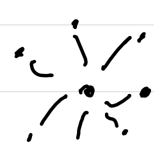
$V_1 + 2V_2 + \dots + mV_m =$

So that $V_1 = V_3 + 3V_4 + \dots + (m-2)V_m + 2$

And thus $|V| = V_2 + 2V_3 + \dots + (m-1)V_m + 2$

$$|E| = |V| + 1.$$

Ex 12.1.10: The maximum value is 31.
Take a spanning tree of G
it will satisfy
$|V| \leq 31$!
We can obtain 31 with



Ex 12.1.16 : (1) We have to remove at one least one edge in the middle triangle and up to three.

If we choose to remove all three we have to remove one edge of the 9-cycle left to get a spanning tree → 9 choices.

If we remove 2 edges in the middle triangle we have to remove one of the 3 edges in the square and one of the 6 of the 7-cycle which gives $3 \times 6 = 18$ choices. But we also have 3 choices of the 2 edges in the triangle which gives $3 \cdot 18 = 54$ possibilities.

If we remove one edge we then have to remove one edge of each remaining square and one of the 5-cycle which gives $3 \times 3 \times 3 \times 3 = 81$ possibilities.

In total we get 144 possibilities.

Ex 12.1.18: We have

$$\sum_v \deg(v) = 2|E|$$

and $|E| = |V| - 1 = 999$

so $\sum_v \deg(v) = 1998$.

Ex 12.2.1: a) k, p, q, s, t, h, f

b) a c) d d) e, f, j, q, s, t

e) q, t f) 2 g) h, p, q, s, t

Ex 12.2.7. (i) We get

a - b - g - c then go to a

a - d - h - e - f.

(ii) h - e - f go back to e

e - a - d go back to a

a - c - g - b

(iii) a - b - g - c go back to a

a - d - h - e - f

Ex 12.2.10: a) The highest height is obtained if the tree is a path with gives height $n-1$.

b) 

The tree is highest if it has this form.

For $2m+1$ vertices we get height m. For n vertices we get height $\lfloor \frac{n-1}{2} \rfloor$.

Ex 12.2.17: a) There are

$1+4+4^2+\dots+4^7$.

b) In the same way:

$1+m+m^2+\dots+m^{h-1}$.