Ex 17.3.1: a) It is enough to make the permutation from the first row, that is: $3 \to 2$ $\quad 1 \to 1$

$4 \to 3$

$2 \to 4$

b) We get the square

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 3 | 4 | 1 | 2 |
| 2 | 1 | 4 | 3 |
| 4 | 3 | 2 | 1 |

c) By reversing the process in part 1 we get the square

| | | | |
|---|---|---|---|
| 1 | 3 | 4 | 2 |
| 4 | 2 | 1 | 3 |
| 3 | 1 | 2 | 4 |
| 2 | 4 | 3 | 1 |

which is orthogonal to the first one.

Ex 17.3.4: It follows from theorem 17.16 that there are $4-1=3$ Latin squares that are $4\times 4$ and orthogonal in pairs. So there are no others.

Ex 17.3.5: If we continue the computations we get

$$L_3 = \begin{matrix} 4 & 5 & 1 & 2 & 3 \\ 2 & 3 & 4 & 5 & 1 \\ 5 & 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 1 & 2 \\ 1 & 2 & 3 & 4 & 5 \end{matrix} \qquad L_4 = \begin{matrix} 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$$

To obtain the standard form we perform the following permutations.

| $L_1 \to L_1'$ | $L_2 \to L_2'$ | $L_3 \to L_3'$ | $L_4 \to L_4'$ |
|---|---|---|---|
| $1 \to 5$ | $1 \to 4$ | $1 \to 3$ | $1 \to 2$ |
| $2 \to 1$ | $2 \to 5$ | $2 \to 4$ | $2 \to 3$ |
| $3 \to 2$ | $3 \to 1$ | $3 \to 5$ | $3 \to 4$ |
| $4 \to 3$ | $4 \to 2$ | $4 \to 1$ | $4 \to 5$ |
| $5 \to 4$ | $5 \to 3$ | $5 \to 2$ | $5 \to 1$ |

Ex 17.4.3: We have $\mathbb{Z}_3 = \{0,1,2\}$ so there are $3^2 = 9$ points and $3^2+3 = 12$ lines in 4 parallel classes given by the slopes 0, 1, 2 and $\infty$.

Slope 0: $y=0$, $y=1$, $y=2$

Slope 1: $y = x$, $y = x+1$, $y = x+2$
①  ②  ③

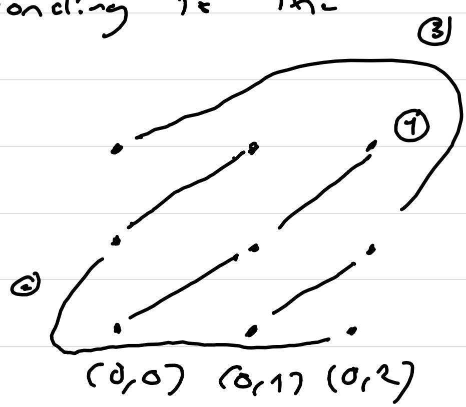
Slope 2: $y = 2x$, $y = 2x+1$, $y = 2x+2$

Slope $\infty$: $x=0$, $x=1$, $x=2$

The latin square corresponding to the class of slope 1 is:

| | | |
|---|---|---|
| 3 | 2 | 1 |
| 2 | 1 | 3 |
| 1 | 3 | 2 |

from



For the class of slope 2 we get

3 1 2

2 3 1

1 2 3

Ex 17.4.5: The equations for the lines are as follows:

a) $y = 4x + 1$

b) $2x + 3y + 3 = 0$

c) $10y = 11x$

Ex 17.4.6: a) The conditions (A1) and (A2) fail: (A1) because $(2,4)$ and $(5,4)$ are both on $y = 2x$ and $y = 4x + 2$

(A2) because $(2,4)$ does not lie on $y = 3$, it contradicts the uniqueness of a line containing $(2,4)$ and not intersecting $y = 3$ since $y = 2x$ and $y = 4x + 2$ check both conditions.

b) There are 36 points and 42 lines. Each line contains 6 points and each point is on 7 lines.