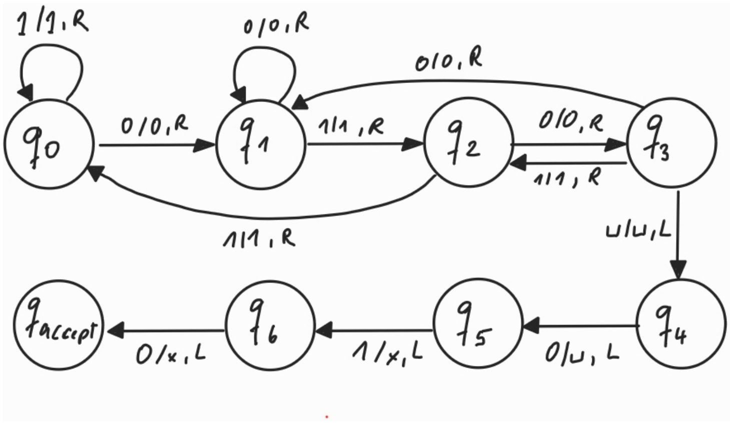
Stockholm University
Department of Mathematics
Lecturer: Marc Hellmuth
Tutor: Anna Lindeberg and Liam Cuclair

1. Exercise "Algorithm and Complexity (DA 4005)"

This exercise counts as a part of Individual Assignment 1

Exercise 1: Turing Machine 1.5+1.5+3=6
Given is the following (overly complicated designed) Turing machine M with initial state q_0 and blank symbol "␣" (in simplified finite state representation).



(a) Specify for the input string $s = 001010$ the string together with the position of the read/write head for each step the string is processed and provide the final string after M reaches the accepting state.

(b) Which language does M recognize in general? Explain your result.

(c) As this TM is overly complicated, find an equivalent TM M' that has as few as possible states (in simplified finite state representation). Explain your Turing machine M' by using the example $s = 001010$.

Exercise 2: Graphs (1.5+1.5)+(1.5+1.5)=6
Let $G = (V, E)$ and $H = (W, F)$ be two undirected graphs.

(a) Let $V = \{0, 1, 2, a, b, c\}$ and $E = \{\{0, 1\}, \{a, b\}, \{1, 2\}, \{b, c\}, \{0, 2\}, \{2, a\}, \{a, c\}\}$ and $W = \{x, y, z, a, b, c\}$ and $F = \{\{a, c\}, \{x, z\}, \{a, b\}, \{z, y\}, \{x, y\}, \{x, a\}, \{b, c\}\}$.

(i) Show that H and G are isomorphic. Explain, how many isomorphism are there?

(ii) How many spanning trees does G have?

(b) Recall: the degree $\deg(v)$ of a vertex v is the size $|N(v)|$ of the set of neighbors of v.

(i) Show that every graph contains always two vertices of the same degree.

(ii) Assume that G is k-regular, i.e., $\deg(v) = k$ for all $v \in V$.
Show that G contains a simple path of length k.

Exercise 3: Recursion 7

Give a pseudocode of a recursive algorithm to compute the coefficients of the polynomial

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

Proof the correctness of your algorithm.

Exercise 4: Runtime 1.5+(1.5+1.5+1.5)=6

(a) Determine the runtime of the following pseudocode in "big-O" notation

for (**int** $i = 1; i < n; i = 2i$) **do**
    for (**int** $j = n; j > 1; j = \frac{j}{2}$) **do**
        for (**int** $k = j; k < n; k = k + 2$) **do**
            print "Hello World!"

(b) Show for

(i) $f(n) = \log_2(n)$ and $g(n) = \log_8(n)$

(ii) $f(n) = \log_2(n^{\log_2(17)})$ and $g(n) = \log_2(17^{\log_2(n)})$

(iii) $f(n) = 8^n$ and $g(n) = 4^n$

which of the relationships $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$ and $f(n) \in \Theta(g(n))$ are satisfied and which are not.

Deadline: Sunday - September 14, 2025