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4. Exercise "Algorithm and Complexity (DA 4005)"

This exercise counts as a part of Individual Assignment 2

Exercise 1: Approximation Algorithm 5+3=8

Let $S = \{1, \ldots, n\}$ be a finite set and $C \subseteq \mathbb{P}(S)$ a collection of subsets of S. Every element $A \in C$ contains at most 3 elements, that is, $|A| \leq 3$. A HittingSet for C is a subset $X \subseteq S$ such that for every element $A \in C$ it holds that $A \cap X \neq \emptyset$. The aim is to find a HittingSet of minimum size, which is an NP-hard problem.

The following algorithm tries to approximate a solution.

HittingSet(S, C)

$X \leftarrow \emptyset$
while $C \neq \emptyset$ **do**
  $A \leftarrow$ arbitary element in C
  $X \leftarrow X \cup A$
  $C \leftarrow C \setminus \{A \in C \mid A \cap X \neq \emptyset\}$
return X

(a) Show that algorithm HittingSet is a 3-approximation algorithm, that is, if I is an instance of HittingSet and $A(I)$ the size of a hitting set returned by HittingSet with input I and $OPT(I)$ the size of an optimal hitting set for I then $A(I) \leq 3 \cdot OPT(I)$.

(b) Show that, in general, there is no constant ρ with $1 \leq \rho < 3$ such that algorithm HittingSet is a ρ-approximation algorithm.

Exercise 2: FPT for the Zombie Apocalypse Defense Problem 4

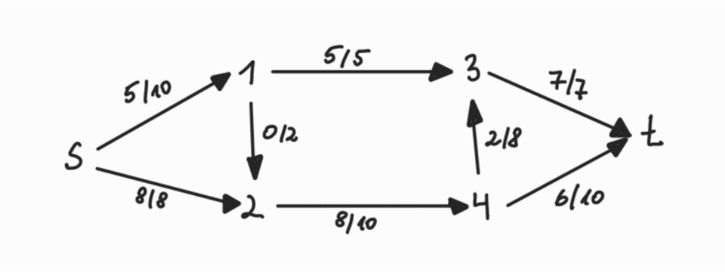
Your city is under a sudden zombie outbreak! The city consists of n hubs connected by streets. Two hubs are neighbors if they are connected by a street. To defend the city, a team of zombie-fighting squads $\mathcal{S}$ can be stationed at some of the hubs.

Each squad $s \in \mathcal{S}$ can protect the hub h where it is stationed, as well as all neighboring hubs of h. A city is considered *protected* if all hubs are protected. The goal is to protect the entire city using at most k squads.

Show that this problem is FPT if each hub has at most 3 neighboring hubs.

Exercise 3: Maximum Flow 2+4+2=8

Given is the following flow network $G = (V, E)$ together with potential flows / capacities along its edges. Non-edges (u, v) have capacity $c(u, v) = 0$.



(a) Let $f\colon V \times V \to \mathbb{R}$ be specified as shown in the figure and where are all non-edges (u, v) receive value $f(u, v) = 0$. Show that f is a flow for G.

(b) Suppose that these values f have already been computed with the Ford-Fulkerson-Algorithm. Continue now with this algorithm to compute a maximum flow in G. Draw for each individual step the residual network.

(c) Determine a minimum cut in G. Is this minimum cut unique?

Deadline: Friday - October 17