Stockholm University Department of Mathematics

Lecturer: Marc Hellmuth

Tutor: Anna Lindeberg and Liam Cuclair

1. Exercise "Algorithm and Complexity (DA 4005)"

This exercise counts as a part of the Practical Exercises

Exercise 1: Depth-First-Search (DFS)

Read Chapter 22.3 Depth-first search and 22.4 Topological sort in "Introduction to Algorithms", Cormen et al., to get familiar with the following algorithm:

```
DFS(G)
                                                       DFS-Visit(v)
1: for each vertex v of G do
                                                         1: v.discoverTime \leftarrow t \leftarrow t + 1
2:
       v.color \leftarrow \text{white}
                                                         2: v.color \leftarrow gray
       v.pred \leftarrow NULL
3:
                                                         3: for each w in the adjacency list of v do
4: t \leftarrow 0
                                                                 if w.color = white then
5: for each vertex v of G do
                                                                     w.pred \leftarrow v
                                                         5:
       if v.color = white then
                                                                     DFS-VISIT(w)
6:
                                                         6:
            DFS-Visit(v)
7:
                                                         7: v.finishTime \leftarrow t \leftarrow t + 1
                                                         8: v.color \leftarrow black
```

In particular get familiar with back edges and the characterization of acyclic graph (Lemma 22.11).

Implement the depth-first-search (DFS) algorithm for directed graphs in C++, Java, or Python. Use this algorithm to decide, whether a given directed graph G = (V, E) is cyclic.

Answer the following question doing simulations:

What is the approximate probability, that a random directed graph with n = 10 nodes is cyclic, when for any two vertices nodes $u, v \ (u \neq v)$ there is an edge from u to v with probability p = 0.15, independent of all other edges? Self-loops $(u, u) \in E$ are not allowed. Simulate at least m = 1000 graphs and report the relative frequency of cyclic graphs. Turn in your code and the output of your program (incl. answer for the latter question).

When handing in programming exercises, always document how to compile and run your program. Comment your source code and do not copy from WWW!

Deadline: Wednesday - Sept 17, 2023