STOCKHOLM UNIVERSITY
Department of Mathematics
Examiner: Gregory Arone

Examination for
MM3001: Matematiska metoder för ekonomer
14th April 2021

Time: 8:00-13:00
Instructions:
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear solutions even if the answer is correct.
- Use natural language when appropriate, not just mathematical symbols.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting $\boxed{\text{A BOX}}$ around it.
- The solutions should be uploaded onto the course's webpage no later than 13:30

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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1. (a) Suppose f is the following function

$$f(x) = \int_0^x e^{t^2}\, dt. \qquad \text{(2p)}$$

What is $f'(x)$? Hint: do not try to evaluate the integral.

(b) Now suppose g is the following function (defined for $x > 0$)

$$g(x) = \int_0^{\sqrt{x}} e^{t^2}\, dt \qquad \text{(3p)}$$

What is $g'(x)$?

2. Suppose that $f(x), g(x)$ are two functions, and $h(x) = f(g(x))$. Assume furthermore that

$$f(2) = 4, \quad f'(2) = -2, \quad g(3) = 2, \quad g'(3) = -1.$$

(a) What is the equation of the tangent line to the graph of f at the point $(2, f(2))$? (2p)

(b) What is $h(3)$? (1p)

(c) What is $h'(3)$? (2p)

3. (a) Compute the improper integral $\displaystyle\int_{e^2}^{\infty} \frac{1}{x(\ln x)^2}\, dx$. (3p)

(b) Let a be a fixed number, and let $f(x)$ be the following function, depending on a

$$f(x) = \begin{cases} \sqrt{ax} & x > 2 \\ a - x & x \leq 2 \end{cases}$$

For which value(s) of a is this function continuous? (2p).

4. Consider the function of two variables: $f(x,y) = x^2y - xy$
   (a) Find the critical points of this function. (3p)
   (b) For each one of the critical points, determine if it is a local maximum, a local minimum, or neither. (2p)

5. Consider the function
$$f(x) = \sqrt{x^2 + x + 2}.$$
   (a) Find the domain of definition of f. (1p)
   (b) Find the intervals where f is increasing and where f is decreasing. (2p)
   (c) Find the minimum and the maximum of f on the interval $[-5, 5]$. (2p)

6. Let a be a fixed number. Consider the following system of equations
$$\begin{aligned} x + 2y + 3z &= 2 \\ x - y + 2z &= 1 \\ x + (10-a)y + 5z &= 3 \end{aligned}$$
   Use Gaussian elimination to:
   (a) Find for which values of a (if any) the system has a unique solution, for which it has no solutions and for which it has infinitely many solutions. (2p)
   (b) In cases when there is a unique solution, express the solution in terms of a. (3p)

Formulas

The second derivative criterion for a function of two variables $f(x,y)$ depends on the determinant $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix}$. It says the following: If, at a critical point

- $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix} > 0$ and $f''_{xx} > 0$ then f has a local minimum at this critical point.
- $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix} > 0$ and $f''_{xx} < 0$ then f has a local maximum at this critical point.
- $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix} < 0$ then f has neither a local maximum nor a local minimum at this critical point.

GOOD LUCK!