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Solutions to the examination paper in Mathematics for Economic and Statistical Analysis, Master Programme, October, 22, 2008

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1. This geometric serie with $r = \frac{3}{5}e^x$ is convergent if $-1 < r < 1$, i.e. only if $\frac{3}{5}e^x < 1$. Thus $e^x < \frac{5}{3}$, and we have $x < \ln\frac{5}{3}$.
The sum S equals $\frac{1}{1-\frac{3}{5}e^x}$, hence, if $S = 5$ then $\frac{1}{1-\frac{3}{5}e^x} = 5$. This gives $e^x = \frac{4}{3}$ and then $x = \ln\frac{4}{3}$.

2. We are looking for the line $y = ax + b$, with slope $a = f'(0)$. Since $f'(x) = 4x + 2/(1+2x)$ then we get $a = 2$. Thus we have $y = 2x + b$. In order to find b we use the fact that the point $(0, 3)$ lies on this line, i.e. $-3 = 2 \cdot 0 + b$. Hence $b = -3$ and $y = 2x - 3$.

3. In order to find the stationary points we solve the system of equations $f'_x = 0$ and $f'_y = 0$, i.e. $3x^2 + 3y = 0$ and $3y^2 + 3x = 0$. The system $x^2 + y = 0$ and $y^2 + x = 0$ can be solved by for example substitution $y = -x^2$ giving $x^4 + x = 0$, i.e. $x(x^3 + 1) = 0$. Thus $x = 0$ or $x = -1$. We get two stationary points: $(0, 0)$ and $(-1, -1)$.
Now we find that $A = f''_{xx} = 6x$, $B = f''_{xy} = 3$ and $C = f''_{yy} = 6y$.
For $(0, 0)$ we get $AC - B^2 = -9 < 0$, which means that $(0, 0)$ is a saddle point. For $(-1, -1)$ we get $AC - B^2 = 27$, while $A = -6 < 0$, which means that $(-1, -1)$ is a local maximum.

4. a) The substiution $x^2 + 2 = t$, $2dx = dt$, and the fact that when x varies between 0 and 1 the t varies between 2 and 3, leads us to the integral $\int_2^3 \frac{1}{2}e^t\,dt = \frac{1}{2}e^t\Big|_2^3 = \frac{1}{2}(e^3 - e^2) = \frac{e^2}{2}(e-1)$.
b) Integration by parts gives $\int y^{1/2}\ln y\,dy = \frac{2}{3}y^{3/2}\ln y - \int \frac{2}{3}y^{3/2}\frac{1}{y}\,dy = \frac{2}{3}y^{3/2}\ln y - \frac{2}{3}\int y^{1/2}\,dy = \frac{2}{3}y^{3/2}\ln y - \frac{2}{3}\cdot\frac{2}{3}y^{3/2} + C = \frac{2}{3}y^{3/2}\big(\ln y - \frac{2}{3}\big) + C$.

5. The implicit differentiation gives $2xy^3 + x^2 3y^2 y' + 10x^4 y + 2x^5 y' - 3y' - 12 = 0$, which can be written as $(3x^2y^2 + 2x^5 - 3)y' = 12 - 2xy^3 - 10x^4 y$. Thus $y'(x) = \frac{12 - 2xy^3 - 10x^4y}{3x^2y^2 + 2x^5 - 3}$. Substitution of $x = -1$ and $y = 1$ gives finally $y' = \frac{12 + 2 - 10}{3 - 2 - 3} = -2$

6. The standard calculations of determinants gives
$\begin{vmatrix} 1 & -1 & 2 \\ 1 & -2 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 4$, $\begin{vmatrix} 3 & -1 & 2 \\ 4 & -2 & 1 \\ 5 & -1 & 1 \end{vmatrix} = 8$, $\begin{vmatrix} 1 & 3 & 2 \\ 1 & 4 & 1 \\ 2 & 5 & 1 \end{vmatrix} = -4$ and $\begin{vmatrix} 1 & -1 & 3 \\ 1 & -2 & 4 \\ 2 & -1 & 5 \end{vmatrix} = 0$. From this we get $x = \frac{8}{4} = 2$, $y = \frac{-4}{4} = -1$ and $z = \frac{0}{4} = 0$.

Paul