Department of Mathematics
STOCKHOLM UNIVERSITY

Examiner: Paul Vaderlind

Examination in
Mathematics for Economic and Statistical Analysis
Master Program; 7,5 ECTS
January, 11, 2010; time: 09:00–14:00

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. It is known that $x > 0$, $y > 0$ and $x + y < 3$. With those conditions, for which points (x, y) in the plane is the series $\sum_{n=0}^{\infty} \frac{x^n + y^n}{(xy)^{n+2}}$ convergent.

2. Evaluate the following integrals:
a) $\int_1^e (\ln x)^2 \, dx$, b) $\int \frac{4x + 10}{\sqrt[3]{x^2 + 5x + 2}} \, dx$

3. (Implicit differentiation) Find the equation for the tangent line to the curve $x^3y^3 - 2x^2y^2 - 4y + x^2 - 2 = 0$ at the point $(1, -1)$. (Consider y as a function of x.)

4. The line $y = 12x + 5$ is a tangent line to the graph of the function $f(x) = 2x^3 - 3x^2 + 25$ at some point $x = a$. Find a.

5. Find the numbers x, y and z satisfying the matrix equality

$$\begin{pmatrix} 3 & 6 & 1 \\ -1 & 2 & -3 \\ 4 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & -5 & 2 \\ 3 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} y \\ z \\ -3 \end{pmatrix}$$

6. Let $f(x) = 6xe^{\frac{x^2 - 10x}{12}}$. Find the intervals where the function is increasing and decreasing, find local extreme points, global extreme points (if they exist) and points where the function crosses the coordinate axes. Sketch the graph of the function.

7. Find the maximum and the minimum value of the function $f(x, y) = xy - x - y + 3$ in the triangle with the vertices at $(0, 0)$, $(2, 0)$ and $(0, 4)$.

GOOD LUCK!

The papers will be handed out at 12.00 on January 20, 2010, in the room next to the Coffee Shop, house 5, and after that in room 208, house 6.