

Examiner: Paul Vaderlind

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

1. For which x is the series $\sum_{n=0}^{\infty} \frac{1}{(1+x)^n}$ convergent?

2. Evaluate the following integrals:

a) $\int_1^e \frac{1 - \ln x}{x^2} dx$, b) $\int \frac{1}{t \ln t} dt$.

3. (Implicit differentiation.) The expression $e^{x^2+y} + y \ln x = x^2$ defines y as a function of x . Is this curve convex or concave at the point $x = 1$?

4. For which values of the constant c the system of equations below has a unique solution, infinitely many solutions and no solutions at all?

$$\begin{cases} x + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y + (c^2 - 14)z = c + 2 \end{cases}$$

5. Consider the function $f(x) = \frac{e^x}{x}$. In which intervals is this function increasing, decreasing, convex and concave? Identify the local extremum points. Are there any global extremum points?

6. Find all stationary points of the function $f(x, y) = 3x^2 + 3xy + y^2 + y^3$ and determine whether they are max-, min- or saddlepoints.

7. Let $f(x, y) = (2x + y)e^{x - \frac{1}{2}y}$. There is a number a such that $f''_{xx} - 4f''_{yy} = a(f'_x + 2f'_y)$. Find a .

GOOD LUCK!

The papers will be handed out at 12.00 on Wednesday, November 10, 2010, in the room next to the Coffee Shop, house 5, and after that in room 208, house 6.