

Examiner: Paul Vaderlind

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

1. For which x is the series $\sum_{n=0}^{\infty} (1+x)^{2n}$ convergent?

2. Evaluate the following integrals:

a) $\int_0^1 (x^3 + x) \ln(x^2 + 1)^2 dx,$ b) $\int t^2 \sqrt{t+1} dt.$

3. (Implicit differentiation.) The expression $xy^2 + y \ln(x+y) = e^x - 1$ defines y as a function of x . Consider the tangent line to this curve at the point $x = 0$. At which point(s) does this line cut the parabola $y = x^2 - 2x - 1$?

4. Solve the equation $\begin{vmatrix} -1 & 2 & x \\ x & 1 & 0 \\ 2 & x & -1 \end{vmatrix} + \begin{vmatrix} 1 & x & 1 \\ 2 & 3 & x \\ 2 & -1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 3 & -2 \\ 5 & x & 1 \\ 2 & -1 & 1 \end{vmatrix}.$

5. In which intervals is the function $f(x) = (x^2 - 3)e^x$ increasing, decreasing, convex and concave? Identify the local extremum points. Are there any global extremum points?

6. Let $f(x, y) = xy^2 - x^2y$. Find the largest and the smallest value of this function on the square with vertices at $(0, 0)$, $(1, 0)$, $(0, 1)$ and $(1, 1)$.

7. Let $f(x, y) = \sqrt{xy} - \frac{1}{2}y(\ln x + \ln y)$. Find $x^2 f''_{xx} - y^2 f''_{yy} + 2xy f''_{xy}$.

GOOD LUCK!

The papers will be handed out at 12.00 on Wednesday Dec 22, 2010, in the room next to the Coffee Shop, house 5, and after that in room 208, house 6.