Department of Mathematics
STOCKHOLM UNIVERSITY

Examiner: Paul Vaderlind

Examination in
Mathematics for Economic and Statistical Analysis
Master Program; 7,5 ECTS
October 22, 2011; time: 09:00–14:00

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. The tangent line to $f(x) = 6x^4 - 40x^3 + 51x^2 - 24x + 7$ is parallel to $x-$ axis at some integer point $x = a$. Find this point and determine whether the function is convex or concave at this point.

2. a) Evaluate the following integral: $\int (1 + e^x) \ln(e^x + x) dx$.

b) Determine if the improper (generalized) integral $\int_0^{0,5} \frac{1}{x \ln^2 x}\, dx$ exists and evaluate it if it does.

3. (Implicit differentiation.) The expression $e^y x^2 + e^x y^3 - x^3 + 8 = 4x$ defines y as a function of x. What is the equation of the tangent line at the point $x = 0$?

4. Find all stationary points of the function $f(x, y) = x^3 - 12x + y^3 + 3y^2 - 9y$ and determine whether they are max-, min- or saddlepoints.

5. Let $h(x, y) = \ln \sqrt{x^2 + y^2}$. Find $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}$ and $-\frac{(x^2 + y^2)^2}{2} \cdot \frac{\partial^2 h}{\partial x \partial y}$.

6. For which x is the series $\sum_{n=0}^{\infty} (\frac{2}{x^2 + 1})^n$ convergent and when does the sum equal 3?

7. Among all lines tangent to the graph of $f(x) = \frac{6}{x^2 + 3}$, find the equations of tangent lines of minimum slope and maximum slope. (Hint: Find first the general equation for a slope of tangent lines to this curve.)

GOOD LUCK!

The papers will be handed out at 12.00 on Thursday, November 3, 2011, in the room next to the Coffee Shop, house 5, and after that in room 208, house 6.