

Examiner: Paul Vaderlind

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

1. Let $f(x) = ax^3 + bx^2 + c$. It is known that the line $y = -3x + 2$ is a tangent to $f(x)$ at $x = -1$ and that $f(x)$ has an inflexionspoint at $x = 1$. Find a , b and c .

2. a) Evaluate the following integral: $\int \frac{\ln x \sqrt{\ln x + 1}}{x} dx$,

b) Determine if the improper (generalized) integral $\int_0^{\infty} \frac{e^x}{\sqrt{e^x + 2012}} dx$ exists and evaluate it if it does.

3. (Implicit differentiation.) The expression $x^2 \ln(xy) + 1 = e^{x^2 + y^2 - 2}$ defines y as a function of x . Determine the points where the tangent line at $(x, y) = (1, 1)$ passes through the coordinate axes.

4. Let $A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$ and let E be a 3×3 unit matrix. Find all real numbers λ which satisfy the equation $\det(A - \lambda E) = 0$.

5. Let $f(x) = (3 - 2x)e^{x^2}$. Find the intervals where the function is increasing and decreasing, find local extreme points, global extreme points (if they exist) and points where the function crosses the coordinate axes. Sketch the graph of the function.

6. Let $g(x, y) = xe^y + ye^x$. Find the sum $\frac{\partial^3 g}{\partial x^3} + \frac{\partial^3 g}{\partial y^3} - x \frac{\partial^3 g}{\partial x \partial y^2} - y \frac{\partial^3 g}{\partial x^2 \partial y}$.

7. Consider the function $f(x, y) = x^2 + 3y^2 - 4x - 6y$. Find the maximum value and the minimum value of this function on the triangle with vertices at $(0, 0)$, $(4, 0)$ and $(0, 3)$.

GOOD LUCK!

The papers will be handed out at 11.00 on Wednesday, January 11, 2012, in the room next to the Coffee Shop, house 5, and after that in room 208, house 6.