

Examiner: Paul Vaderlind

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

1. For which x is the series $\sum_{n=0}^{\infty} \left(\frac{x^2 - 1}{3}\right)^n$ convergent. For what values of x is the sum 3?

2. Determine if the given improper integrals exist, and evaluate them if they do:

a) $\int_0^3 \frac{x}{\sqrt{9-x^2}} dx,$ b) $\int_0^{\infty} \frac{dz}{\sqrt{z+1}}.$

3. (Implicit differentiation) Find the equation for the tangent line to the curve defined by

$$2x^3e^y - y^3e^x - \ln(x(y+1)) + x + y - 3 = 0$$

at the point $(1, 0)$. (Consider y as a function of x .)

4. Find a and b such that the function $f(x) = \frac{ax^3 + bx - 3b}{x + 1}$ has a local extreme at the point $(1, 7)$. Is this local extreme a local max- or a local min-point?

5. Find all z such that $\begin{vmatrix} z & -2 & -1 \\ 1 & 1 & z \\ 1 & z & 2 \end{vmatrix} = \begin{vmatrix} z & 0 & -1 \\ -7 & z & 1 \\ -3 & 1 & -4 \end{vmatrix}.$

6. For which points (x, y) is the function $g(x, y) = \frac{\ln x}{\ln(1-x)} + \frac{\ln(1-y)}{\ln y}$ well-defined? Does the equation $g(x, y) = -2$ have a solution? Does the equation $g(x, y) = 2$ have a solution? (It is enough to either give one solution or to prove that there are no solutions at all.)

7. Let $h(x, y) = 5 - 3y + 3x^2y + 2(x^3 + y^3)$. Find all stationary points for this function and decide whether they are local max-, min-, or saddle-points.

GOOD LUCK!

The papers will be handed out on Tuesday, Nov 13, 2012, at 12.30, in the room next to the Coffee Shop, house 5, and after that in room 204, house 6.