

Examiner: Paul Vaderlind

No calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

1. What region in the plane is described by the system of inequalities below?

$$\begin{cases} x + 2y + 1 \geq 0 \\ 5x + 3y + 19 \geq 0 \\ -2x + 3y + 5 \geq 0 \end{cases}$$

Draw a picture and mark important points.

2. Evaluate the following integrals:

a) $\int_1^2 x(\ln x)^2 dx$, b) $\int \frac{e^x + 1}{(e^x + x + 1)^3} dx$

3. (Implicit differentiation) The curve $2x^3y^2 + 3x^2y^3 - x^2y + y^2 + xy = 0$ passes through a point $(x_0, 1)$, for some real number x_0 . Find the equation for the tangent line to the curve at this particular point. (Find x_0 and consider y as a function of x .)

4. Solve the equation $4x^3 - 2x^2 - 4x - 1 = 0$.

5. Find the numbers x , y and z satisfying the matrix equality

$$\begin{pmatrix} 2 & 3 & 1 \\ -3 & 0 & 2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & -3 \\ 2 & 1 & 6 \\ 3 & 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ -1 \\ z \end{pmatrix} + \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & -1 \\ 2 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

6. Let $f(x) = (2x^2 - 1)e^{4x+1}$. Find the intervals where the function is increasing and the intervals where the function is increasing decreasing, its local extreme points, its global extreme points (if they exist) and the points where the function crosses the coordinate axes. Sketch the graph of the function.

7. Find all stationary points for the function $f(x, y) = ye^{x^2 - \frac{1}{2}y^2}$ and determine whether they are maximum, minimum or saddle points.
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GOOD LUCK!

The papers will be handed out on Monday, Jan 14, 2013, at 12.00, in the room next to the Coffee Shop, house 5, and after that in room 204, house 6.