Only non graphic calculators are allowed.
Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

1. Find the equation of the tangent line to the graph of the function $f(x) = e^{x^2-1} + x\ln(2x-1) + x^3$ at the point $(x, y) = (1, 2)$.

2. Find the Taylor polynomial of degree 3 at $a = 0$ (MacLaurin polynomial) for the function $f(x) = e^{2x} + \ln(x+1)$.

3. Consider two series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{(a+1)^2}\right)^n$ and $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{a^2+2}\right)^n$

a) For what values of a are **both** series convergent?

b) For what values of a do they have the same sum?

4. Find all real numbers x such that $\begin{vmatrix} -2 & x & -2 \\ -1 & \frac{1}{2} & 2x \\ x & -x & 1 \end{vmatrix} = \begin{vmatrix} x & -1 \\ -2 & x \end{vmatrix}$.

5. (a) Find $\displaystyle\lim_{x\to\infty} \frac{x\ln x}{x + \ln x}$.

(b) Evaluate the integral $\displaystyle\int_0^1 (x+1)e^{x^2+2x}\, dx$.

6. (Implicit differentiation) Find numbers a and b such that the line $y = -2x + 1$ is the tangent line to the curve $ax^2y^3 + bx^3y^2 + y + 2 = 0$ (note that y is a function of x: $y = y(x)$), at the point $x = 1$.

7. Let $f(x, y) = 4xy^2 - x^2y^2 - xy^3$. Find the largest and the smallest value of $f(x, y)$ on the triangle whose vertices are $(0, 0)$, $(6, 0)$ and $(0, 6)$.

GOOD LUCK!

The papers will be handed out on Wednesday Oct 30, 2013, at 12.00, in the room next to the Coffee Shop, house 5, and after that in room 204, house 6.