Examination in 'Mathematics for Economic and Statistical Analysis'

Mathematics Department, Stockholm University
Master program 7.5 ECTS.
Date: October 1, 2014. Time 09:00-14:00
Examiner: Maurice Duits

Only non-graphic calculators are allowed. Each solved problem is awarded by up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. It is given that, at $x = 1$, the line $y = bx + 1$ is tangent to the graph of the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = ax^3 + x - 1$. Determine a and b.

2. (a) Compute the integral

$$\int_0^1 (x-1)^2 \mathrm{e}^x \mathrm{d}x$$

   (b) For which value(s) of a does the integral

$$\int_{\mathrm{e}}^{\infty} \frac{1}{x} (\ln x)^a \mathrm{d}x,$$

   converge? Compute the integral for those values.

3. For which value(s) of a does the matrix

$$A = \begin{pmatrix} a & 1 & 2 \\ 0 & 2 & 1 \\ 1 & a & 1 \end{pmatrix}$$

   have an inverse?

4. Determine the Taylor polynomial for $f(x) = \mathrm{e}^{\sqrt{1+x+x^2}}$ of degree 3 around $x = 0$.

5. Let $g(x, y) = x \ln(y - x^2)$.

   (a) Find all points (x, y) for which $g(x, y)$ is well-defined and draw this region in the plane.

   (b) Find all stationary points of g and determine whether they are a local maximum point, local minimal point or a sadle point.

6. (implicit differentation) The equation $y^3 + bxy = 1$ defines y as a function for x for which the graph passes through $(0, 1)$. For which value(s) of b does the tangent line to the graph of the function at $(0, 1)$ go through $(1, 0)$?

7. Find for which values of b, the series

$$\sum_{k=1}^{\infty} \frac{1}{(2b^2 - 1)^k}$$

   converges and compute the value at $b = 2$.

Good luck!

The corrected exams will be handed out on Wednesday, October 8, 10:00, in the room next to the Coffee Shop, house 5, and after that in room 203, house 6.