STOCKHOLM UNIVERSITY
Department of Mathematics
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Examination in
Mathematics for Economic and Statistical Analysis
MM1005, höstterminen 2017, 7.5 ECTS
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Instructions:
- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your chain of reasoning. Use natural language, not just mathematical symbols.
- Write clearly and legibly.
- Mark clearly your final answer to each question by putting $\boxed{\text{a box}}$ around it.

Grades: There are 6 questions. Each solved problem is awarded up to 10 points. At least 30 points are necessary for the grade E, 36 for D, 42 for C, 48 for B and 54 for A. Note that the problems are not ordered according to the difficulty!

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1. Find the following limits.

   (a) $\lim_{t\to 0} \dfrac{1-e^{2t}}{1-e^{t}}$ (b) $\lim_{t\to 0} \dfrac{1-2t}{1-t}$

2. Define $f(x)$ to be the determinant $\begin{vmatrix} 1+x & 1+x \\ x & 4 \end{vmatrix}$.

   (a) $f(x)$ has a global maximum value $f(x_*)$. Find this value.

   (b) Find the limit $\lim_{x\to\infty} f(x)$, or show no limit exists.

3. Find all solutions of the following systems of equations.

   (a)
   $$\begin{aligned} 2x - 2y + w &= 4 \\ x - 3z + w &= 3 \end{aligned}$$

   (b)
   $$\begin{aligned} x_2 + 3x_3 &= 1 \\ x_1 + 3x_3 &= 0 \\ x_1 + 2x_2 + 9x_3 &= 1 \end{aligned}$$

4. Take $f(x,y) = x^3 - 3x - (x+y)^2$.

   (a) Find all stationary points of f, and classify them, i.e. determine whether each one is a maximum, minimum, or saddle point.

   (b) The function f has no maximum on the set $\{(x,y) \in \mathbb{R}^2 \mid x \geq 0\}$. Why does this fact not contradict the Extreme Value Theorem?

5. Consider the level curve defined by $4(U+V)^2 + U - V = 14$.

   (a) Find the points where this curve intersects the V-axis.

   (b) The point $(U_*, V_*) = (\frac{1}{32} + 7, \frac{3}{32} - 7)$ lies on this curve, and near this point, U can be expressed as a function of V. Find the equation of the tangent line to the curve at this point.

6. Find the following integrals.

(a) $\int_0^a xe^{-x^2}\mathrm{d}x$ (b) $\int_0^\infty xe^{-x^2}\mathrm{d}x$ (c) $\int x2^x\mathrm{d}x$

GOOD LUCK! — LYCKA TILL!
