
Instructions:

- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your chain of reasoning. Use natural language, not just mathematical symbols.
- Write clearly and legibly.
- Mark clearly your final answer to each question by putting a box around it.

Grades: There are 6 questions. Each solved problem is awarded up to 10 points. At least 30 points are necessary for the grade E, 36 for D, 42 for C, 48 for B and 54 for A. Note that the problems are not ordered according to the difficulty!

1. Find the following limits.

(a) $\lim_{x \rightarrow 0} \frac{x \ln(1+x)}{3(1+x-e^x)}$

(b) $\lim_{x \rightarrow \infty} \frac{50x^2}{x^3 - x + 1}$

2. Suppose a quantity P is a function of another quantity T , given by $P = e^T(T^2 - 4T + 1)$. Find the maximum and minimum values that P can take, for T in the range $0 \leq T \leq 4$.
3. Let g be the function given by $g(x, y) = x^3 + y^3 + 3xy + 3$. Find all stationary points of g , and determine whether each is a local maximum, local minimum, or saddle point.
4. Find all numbers a, b, c satisfying the equation

$$\begin{pmatrix} 3 & 6 & 1 \\ -1 & 2 & -3 \\ 4 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & -5 & 2 \\ 3 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} b \\ c \\ -3 \end{pmatrix}$$

5. Find the following integrals:

(a) $\int_0^1 ye^{y^2-1} dy$

(b) $\int \frac{3 \ln t}{\sqrt{t}} dt$

6. Suppose y is defined implicitly as a function of x by the equation $x^2y^3 + 2x^5y = 3y + 12x + 8$.

- (a) Find the derivative $\frac{dy}{dx}$ at the point $(x, y) = (-1, 1)$.
- (b) Find all values of x such that $y = 0$.

GOOD LUCK! — LYCKA TILL!
