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**Instructions:**

- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your chain of reasoning. Use natural language, not just mathematical symbols.
- Write clearly and legibly.
- Mark clearly your final answer to each question by putting a box around it.

**Grades:** There are 6 questions. Each solved problem is awarded up to 10 points. At least 30 points are necessary for the grade E, 36 for D, 42 for C, 48 for B and 54 for A. Note that the problems are not ordered according to the difficulty!

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1. Find the following limits.

(a)  $\lim_{x \rightarrow 0} \frac{x \ln(1+x)}{3(1+x-e^x)}$

(b)  $\lim_{x \rightarrow \infty} \frac{50x^2}{x^3 - x + 1}$

2. Suppose a quantity  $P$  is a function of another quantity  $T$ , given by  $P = e^T(T^2 - 4T + 1)$ . Find the maximum and minimum values that  $P$  can take, for  $T$  in the range  $0 \leq T \leq 4$ .
3. Let  $g$  be the function given by  $g(x, y) = x^3 + y^3 + 3xy + 3$ . Find all stationary points of  $g$ , and determine whether each is a local maximum, local minimum, or saddle point.
4. Find all numbers  $a, b, c$  satisfying the equation

$$\begin{pmatrix} 3 & 6 & 1 \\ -1 & 2 & -3 \\ 4 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & -5 & 2 \\ 3 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} b \\ c \\ -3 \end{pmatrix}$$

5. Find the following integrals:

(a)  $\int_0^1 ye^{y^2-1} dy$

(b)  $\int \frac{3 \ln t}{\sqrt{t}} dt$

6. Suppose  $y$  is defined implicitly as a function of  $x$  by the equation  $x^2y^3 + 2x^5y = 3y + 12x + 8$ .

- (a) Find the derivative  $\frac{dy}{dx}$  at the point  $(x, y) = (-1, 1)$ .
- (b) Find all values of  $x$  such that  $y = 0$ .

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**GOOD LUCK! — LYCKA TILL!**

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