STOCKHOLM UNIVERSITY
Department of Mathematics
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Examination in
Mathematics for Economic and Statistical Analysis
MM1005, höstterminen 2018, 7.5 ECTS
Wednesday 3 October 2018

Instructions:
- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your chain of reasoning. Use ordinary language, not just mathematical symbols.
- Write clearly and legibly.
- Mark your final answer to each question clearly by putting $\boxed{\text{a box}}$ around it.

Grades: There are 6 questions. Each solved problem is awarded up to 10 points. At least 30 points are necessary for the grade E, 36 for D, 42 for C, 48 for B and 54 for A. Note that the problems are not ordered according to difficulty!

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1. Find the following limits.

   (a) $\lim_{x\to 2} \dfrac{3^x - x^2}{x^2 - 3}$ (b) $\lim_{x\to 2} \dfrac{2^x - x^2}{x^2 - 4}$ (c) $\lim_{x\to 2^+} \dfrac{3^x - x^2}{x^2 - 4}$

2. Consider the function $f(x) = \frac{x^2+2}{x}$, on the interval $[\frac{1}{2}, 3]$.

   (a) Find the maximum and minimum values of f on this interval.

   (b) For which values of x is $f(x)$ decreasing/increasing, within the given interval?

3. Find all solutions to the following systems of equations:

   (a)
   $$\begin{aligned} 6a &+ 2c = 6 \\ -3a &- 2c = 0 \\ a - b - c &= 2 \end{aligned}$$

   (b)
   $$\begin{aligned} x &+ 2z &&= -3 \\ -2y &+ 2w &&= 0 \\ -2x + y - 4z - w &&&= 6 \end{aligned}$$

4. Show that for all $x \geq 0$,

   $$1 + x - \frac{1}{2}x^2 \quad \leq \quad \sqrt{1+2x} \quad \leq \quad 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3.$$

   Hint: take a second-order Taylor expansion of $\sqrt{1+2x}$, and bound the error term.

5. Find the following integrals:

   (a) $\displaystyle\int t^2(\ln 2t)\, \mathrm{d}t$ (b) $\displaystyle\int_1^e \frac{\ln y}{y((\ln y)^2 + 2)} \mathrm{d}y$

6. Let $g(x, y) = x^2 - 2x + y^2 + C(xy - y)$, where C is some constant.

   (a) Show that for $C \neq 2$, g has a unique critical point.

   (b) Classify for which values of C this critical point is a maximum, minimum, or saddle point.

GOOD LUCK! — LYCKA TILL!