STOCKHOLM UNIVERSITY
Department of Mathematics
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Re-examination in
Mathematics for Economic and Statistical Analysis
MM1005, höstterminen 2018, 7.5 ECTS
Wednesday 7 November 2018

Instructions:
- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your chain of reasoning. Use ordinary language, not just mathematical symbols.
- Write clearly and legibly.
- Mark your final answer to each question clearly by putting $\boxed{\text{a box}}$ around it.

Grades: There are 6 questions. Each solved problem is awarded up to 10 points. At least 30 points are necessary for the grade E, 36 for D, 42 for C, 48 for B and 54 for A. Note that the problems are not ordered according to difficulty!

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1. Find the following limits, if they exist.

   (a) $\lim_{x\to\infty} \frac{x^2}{2-x} + x$ (b) $\lim_{x\to 0} \frac{e^{x^2}-1}{x\ln(2+x)}$

2. Consider the curve given implicitly by the equation $y^3 + x^2y = 10$. (This defines y as a function of x.)

   (a) Find $y(1)$.
   (b) Find the equation of the tangent line to this curve at the point $(1, y(1))$.

3. Evaluate the following integrals, if they exist.

   (a) $\int_1^2 \frac{\sqrt{1+\ln x}}{x}dx$ (b) $\int_0^1 x^2(\ln x)dx$

4. (a) For which values of c is the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & c \\ 1 & c & 1 \end{pmatrix}$ invertible?

   (b) Find all solutions to the equation

   $$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$$

   or determine that it has no solutions.

5. Let $g(x) = \sqrt[3]{1+x}$.

   (a) Find $T_2(x)$, the 2nd-order Taylor approximation of g around $x = 0$.
   (b) Show that the error term $R_3(x) = g(x) - T_2(x)$ satisfies $|R_3(x)| \leq \frac{5x^3}{81}$, for $x \geq 0$.
   (c) Use $T_2(x)$ to compute $\sqrt[3]{(1002)}$ to 7 decimal places. *Hint: use* $1002 = 1000(1 + \frac{2}{1000})$.

6. A quantity C is determined by inputs s, t according to the formula $C = st(s + t - 1)$. Find the maximum and minimum possible values for C, subject to the constraints $s \geq 0$, $t \geq 0$, and $s + t \leq 2$.

GOOD LUCK! — LYCKA TILL!