STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

MT5003
TENTAMEN
090529

Exam in Statistical Inference

090529 kl. 9.00-14.00

Examinator: Åke Svensson, tel. 16 45 69, akes@math.su.se
Aids: Pocket calculator and distributed tables over Normal and χ^2-distributions.
Return: Thursday 4/6 kl 13.00 in room 318, house 6.

Theory: 1

a) Define the score function. (3p)

b) Define the (Fisher-)information. (3p)

c) Formulate the Information inequality. (Proof not needed) (4p)

Theory: 2

a) What are type I and type II errors? (3p)

b) Formulate Neyman-Pearsons lemma (for a simple null-hypothesis against a simple alternative). (Proof not needed) (4p)

c) Suppose the alternative is composite. What is a uniformly most powerful test? (3p)

Problem: 3

Let $X_1, \ldots, X_n$ be i.i.d observations from a $U(0, \theta)$-distribution, i.e. a uniform distribution on the interval $(0, \theta)$.

a) Verify that $T = X_{(n)}$ is a sufficient statistic. (4p)

b) Derive an unbiased estimator of the parameter θ based on T. (3p)

c) Find the variance of this unbiased estimator. (3p)

Problem: 4

Let $X_1, \ldots, X_n$ be i.i.d observations from a $\mathrm{N}(0, \sigma_0)$-distribution, where σ_0 is assumed to be known.

The UMP test for $\mathrm{H}_0 : \mu = 0$ against the alternatives $\mathrm{H}_\mathrm{A} : \mu > 0$ is to reject the null-hypothesis when $\sum X_i > K$.

a) Find the power function for a test at the risk level 5 % (6p)

b) How many observations is needed to be able to test the alternative $\mu = 1$ with the power 80 %, in the case where $\sigma_0 = 1$? (4p)

Problem: 5

The density of a Beta(s, t) distributed random variable is

$$f(x \mid s, t) = \frac{1}{B(s, t)} x^{s-1}(1 - x)^{t-1}$$

when $0 < x < 1$ where

$$B(s, t) = \int_0^1 x^{s-1}(1 - x)^{t-1} dx.$$

Such a random variable has the mean $\frac{s}{s+t}$ and the variance $\frac{st}{(s+t+1)(s+t)^2}$.

a) Assume that it takes 15 tries in a sequence of independent Bernoulli trials with probability p for success to get the first success. Derive a Bayes - estimate, e.g. the Bayes-estimate with quadratic loss, if the prior distribution is Beta$(10, 4)$ (5p)

b) Prove that you get the same estimate if 1 success is observed in the first 15 trials regardless of when it happens (5p)

Problem: 6

We will compare two different experiments.

In experiment I you perform n independent Bernoulli trials, which each result in success with probability p. Denote the number of successes with X. X has a binomial distribution, $Bin(n, p)$

In experiment II you perform independent Bernoulli trials, which each result in success with probability p until you get m successes. Denote the number trials needed with Y. Y has a negative binomial distribution, $Negbin(m, p)$.

The p.f. of this distribution is:

$$f_Y(y \mid p) = \binom{y-1}{m-1} p^m (1-p)^{y-m}, y = m, m+1, \ldots$$

This distribution has mean m/p and variance $m(1-p)/p^2$.

a) Derive the information in experiment I. (2p)

b) Derive the information in experiment II. (2p)

Assume that both experiments are realized with the result that in experiment I: $X = m$ and in experiment II: $Y = n$. (m is the same number as in experiment II and n the same as in experiment I)

c) Verify that the likelihood functions are the same in both cases. (2p)

d) Assume that asymptotic theory is used to derive confidence intervals for p. Are the intervals different in the two experiments? If they are which one is shortest? (4p)

Lycka till!