STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam:
Econometric methods (MT5014)
2021-02-24

Examiner: Kristoffer Lindensjö
E-mail: kristoffer.lindensjo@math.su.se
Phone: 070 444 10 07

Return of exam: See the course web page.

Information due to this being a home exam:

Complete instructions are found on `https://kurser.math.su.se/course/view.php?id=932` under the headline "HOME EXAM!!!". The following is a summary of the most important instructions:

- This home exam should be handed in on the course webpage (`https://kurser.math.su.se/course/view.php?id=932`) **today (i.e. at the day of the exam) at the latest at 17:00 (deadline)**.
- The solutions to this home exam should be handed in in PDF format (i.e. one PDF file). There are no restrictions regarding what your PDF should contain. For example, the PDF may be based on a Word document, a Latex document, or scanned nicely handwritten solutions. If you plan on "scanning" handwritten solutions using your mobile phone, I suggest downloading and using a "scanning app". If you scan and thereby obtain several PDF files, then there are many programs that can be used to merge several PDF files into one PDF file.
- Write your anonymization-code (anonymiseringskoden) on each page of your solutions. Name your PDF file using your anonymization-code.
- When writing the home exam you may use any literature and computer program.
- Your solutions should be of the same type as for usual exams (i.e. not of "thesis type") .

The exam consists of six problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.

Preliminary grading:

A	B	C	D	E
54	48	42	36	30

Good luck!

Problem 0

The PDF document that contains your home exam should start by you writing the following sentence:

I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person. This means that I have for example not discussed the solutions or the home exam with any other person.

Problem 1

Consider the model

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \quad i = 1, ..., n$$

Let $n = 3$. Suppose

$$V(\varepsilon_1|X_1, X_2, X_3) = V(\varepsilon_2|X_1, X_2, X_3) = V(\varepsilon_3|X_1, X_2, X_3) = 1,$$

$$Cov(\varepsilon_1, \varepsilon_3|X_1, X_2, X_3) = 0$$

and that the classical assumptions are fulfilled with the exception that

$$Cov(\varepsilon_1, \varepsilon_2|X_1, X_2, X_3) = 0.5$$

and

$$Cov(\varepsilon_2, \varepsilon_3|X_1, X_2, X_3) = 0.5.$$

Consider GLS estimation.

Suppose $(X_1, X_2, X_3) = (2, 3, 2)$ and find $Cov(\hat{\alpha}_{GLS}, \hat{\beta}_{GLS}|X_1, X_2, X_3)$ and $Var(\hat{\beta}_{GLS}|X_1, X_2, X_3)$.

Suppose also that $(Y_1, Y_2, Y_3) = (4, 3, 7)$ and find the estimates $\hat{\alpha}_{GLS}$ and $\hat{\beta}_{GLS}$. (10 p)

Problem 2

Consider the model

$$Y_i = \beta X_i + \varepsilon_i, \quad i = 1, ..., n$$

where each X_i and β are one-dimensional constants, $V(\varepsilon_i) = \sigma^2$ and the classical assumptions are fulfilled.

Derive the OLS estimator $\hat{\beta}$ and show whether or not it is unbiased and BLUE?

(10 p)

Problem 3

Consider the model

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + a_i \varepsilon_i$$

where the ε_i satisfy the classical assumptions and the a_i are known constants. It holds that $V(\varepsilon_i) = \sigma^2$, which is a constant that is not known.

You have data corresponding to $(X_{1,1}, X_{1,2}, ..., X_{1,100})$, $(X_{2,1}, X_{2,2}, ..., X_{2,100})$ and $(Y_1, Y_2, ..., Y_{100})$.

Which estimator for $\beta_i, i = 0, 1, 2$, is BLUE?

Based on this estimator, you want to test $H_0 : \beta_1 = \beta_2 = 0$, using an F-test, on the 1% significance level. Describe in detail how this is done.

Hint: your description should for example include: 1. the alternative hypothesis, 2. exactly how the test statistic is calculated (for example, if the formula for the test statistic depends on a parameter n, then you should state what n is in this case and so on) 3. exactly when the null hypothesis can be rejected.

(10 p)

Problem 4

Consider

$$r_t = 0.3r_{t-1} + a_t,$$

where the a_t are IID $N(0, 1)$. Show that $\{r_t\}$ is weakly stationary.

(10 p)

Problem 5

Consider the time series

$$r_t = \begin{cases} 0.4r_{t-1} + a_t & \text{if } r_{t-1} \geq 1, \\ 0.2r_{t-1} + a_t & \text{if } r_{t-1} < 1, \end{cases}$$

where a_t is IID with $Prob(a_t = 1) = Prob(a_t = -1) = \frac{1}{2}$. Suppose $r_1 = 1$. Find the 1 step ahead forecast. Also find the 2 step ahead forecast.

(10 p)

Problem 6

Consider

$$x_t = 0.3x_{t-1} + a_t + 0.5a_{t-1}$$

where $\{a_t\}$ is a white noise time series with $E(a_t) = 0$ and $V(a_t) = 1$.

Find $E(x_t)$.

Express $\{x_t\}$ as an MA process.

(10 p)