STOCKHOLM UNIVERSITY
Department of Mathematics
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Examination in
Mathematics for Economic and Statistical Analysis
MM1005, höstterminen 2019, 7.5 ECTS
Friday 1 November 2019

Instructions:
- During the exam you **may not** use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers — communicate your reasoning. Use ordinary language where appropriate, not just mathematical symbols.
- Write clearly and legibly.
- Mark your final answer to each question clearly by putting $\boxed{\text{a box}}$ around it.

Grades: There are 7 questions. Each solved problem is awarded up to 10 points. At least 35 points are necessary for the grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to difficulty.

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1. Find the following limits:

   (a) $\lim_{x \to \infty} \frac{30x \ln x + 5}{6 - 2x}$ (b) $\lim_{x \to 2} \frac{4}{x \ln x^2}$

2. Take the function f given by $f(x) = x^3 - \frac{3}{2}x^2 - 6x + \frac{9}{2}$.

   (a) Find all critical points of f.
   (b) For which values of x is f increasing/decreasing?
   (c) Find the maximum and minimum values of f on the interval $[0, 3]$.

3. Suppose a quantity K varies over time t according to the formula $K = \sqrt{t^2 - 4t - 4}$.

   (a) Find $\frac{\mathrm{d}K}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2K}{\mathrm{d}t^2}$.
   (b) Give the 2nd-order Taylor approximation of K around time $t = 1$.

4. Find the following integrals:

   (a) $\int 3x \ln x \,\mathrm{d}x$ (b) $\int_1^5 \frac{\ln x}{x} \mathrm{d}x$

5. Find the maximum and minimum values of the function $F(x, y) = 2x^2 + 2y^2 - 4x + 6$ subject to the constraints $x \geq 0$, $x^2 + y^2 \leq 4$.

6. Consider the system of equations

$$\begin{aligned} 2x_1 + 2x_2 \phantom{{}- x_3} &= 7 \\ x_1 + 3x_2 - x_3 &= 9 \\ ax_1 - x_2 + x_3 &= -2 \end{aligned}$$

   (a) Show that for $a \neq 1$, the system has a unique solution.
   (b) For $a = 1$, find the general solution of the system, or show that there are no solutions.

7. Let C be the curve $y = x^3 + 2x - 1$.

   (a) Find the tangent line to C at the point $(-1, -4)$.
   (b) Find all points (x, y) on C such that the tangent to C at (x, y) passes through $(2, 3)$.

GOOD LUCK! — LYCKA TILL!