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solutions in
Kombinatorik 2, 7,5 p
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1a) Generating function $1/(1-x)1/(1-x^2)1/(1-x^3) = 1/(1-x)1/(1-x^2)(1+x^3+x^6+x^9+\cdots) = 1/(1-x)(1+x^2+x^4+x^6+x^8+x^{10}+\cdots)(1+x^3+x^6+x^9+\cdots) = 1/(1-x)(1+x^2+x^3+x^4+2x^6+2x^7+2x^8+2x^9+2x^{10}+\cdots) = (\cdots+14x^{10}+\cdots)$
Answer: 14
b) By transposing the Ferrers diagram, we see that it is equally many, i.e. 14.
c) Generating function $1/(1-x)1/(1-x^3)1/(1-x^5)1/(1-x^7)1/(1-x^9) = \cdots+10x^{10}+\cdots$
Answer: 10
d) Generating function $1/(1-x^2)1/(1-x^4)1/(1-x^6)1/(1-x^8)1/(1-x^{10}) = \cdots+7x^{10}+\cdots$
Answer: 7

2. Let a_n be the number of strings of length n with an odd number of even digits. Then there are $b_n = 3^n - a_n$ strings of length n with an even number of even digits. If the last digit is even, there are $2b_{n-1}$ wanted strings. If the last digit is odd, there are a_{n-1}. Thus $a_n = 2\cdot 3^{n-1} - a_{n-1}$. Now $a_1 = 2$. General solution to the homogeneous equation is $c(-1)^n$. A particular solution is $d\cdot 3^n$. Substituted into the equation gives $d = 1/2$. $a_1 = 2$ gives $c = -1/2$.
Answer: $(-1)^{n+1}/2 + 3^n/2$

3. The polynomial starts with $1+12x$. With 2 rooks:
Row 1 and 2 (or 1 and 3 or 2 and 4 or 3 and 4): 6 possibilities each.
Row 1 and 4: 12 possibilities.
Row 2 and 3: 2 possibilities.
Thus the coefficient of x^2 is 38. With 3 rooks:
Row 1, 2 and 3 (or 2, 3 and 4): 4 possibilities each.
Row 1, 2 and 4 (or 1, 3 and 4): 12 possibilities each.
Thus the coefficient of x^3 is 32.
With 4 rooks there are 4 possibilities.
Answer: $1+12x+38x^2+32x^3+4x^4$.

4. The recurrence equation is $a_n = a_{n-1}+a_{n-2}$, $a_1 = 1, a_2 = 2$. Characteristic equation $r^2-r-1=0$ with roots $(1\pm\sqrt{5})/2$. General solution $a((1+\sqrt{5})/2)^n + b((1-\sqrt{5})/2)^n$. The start conditions give $a = (5+\sqrt{5})/10, b = (5-\sqrt{5})/10$.

5a) $G = (\{a,b,c,d,e\}, \{(a,b),(a,c),(c,d),(c,e),(d,e)\}$ has an Euler circuit $a-c-d-e-b-a$ but no Hamilton cycle since we can't pass c twice.
b) $G = (\{a,b,c,d,e\}, \{(a,b),(a,c),(a,d),(c,e),(d,e)\}$ has a Hamilton cycle $a-b-c-e-d-a$ but no Euler circuit since $\deg(a) = 3$.

6a) $a-c, c-f, f-i, f-h, e-h, e-g, d-g, b-e, g-j$.
b) $a-e-g-j$ of weight 16.

c) The weights are: $a-b$ 8, $a-e$ 5, $a-c$ 6, $b-d$ 4, $b-e$ 4, $c-e$ 3, $c-f$ 3, $d-g$ 2, $d-h$ 2, $e-g$ 3, $e-h$ 3, $e-i$ 6, $f-h$ 2, $f-i$ 1, $g-j$ 5, $h-j$ 7, $i-j$ 7. Value=19. Minimal cut $\{a,b,c,d,e,f,h\} \cup \{g,i,j\}$.