MATEMATISKA INSTITUTIONEN	Tentamensskrivning i
STOCKHOLMS UNIVERSITET	Linjär analys
Avd. Matematik	7,5 hp
Examinator: Sara Maad Sasane	2011-08-12

No calculator is allowed. The problems are not arranged in order of difficulty. Results from a subproblem may be used in a subsequent subproblem even if the earlier one is not attempted. Justify your answers. Grade criteria A: 90–100 points, B: 75–89 points, C: 65–74 points, D: 55–64 points, E: 50–54 points, Fx: 45–49 points, F: 0–44 points. For passing a minimum grade of E is necessary in the written exam and moreover a pass in the compulsory assessed coursework. If a student recieves the grade Fx in the written exam and has completed the assessed coursework in time, then the student will get the possibility to complete another coursework in order to get the grade E instead of Fx.
Good luck!

1. a) Determine the cosine series of the function $f(x) = x$ on the interval $(0, \pi)$. 10 p

   b) Sketch the graph of the function which is represented by this series on the interval $(-5\pi, 5\pi)$. 5 p

2. The function $f(x) = x^4$ has the Fourier series

$$x^4 \sim \frac{\pi^4}{5} + 8\sum_{n=1}^{\infty}(-1)^n \frac{n^2\pi^2 - 6}{n^4}\cos(nx)$$

   on the interval $(-\pi, \pi)$. This may be used without a proof.

   a) Show by referring to a suitable theorem that the series above converges (to some function) on the whole real line. 10 p

   b) Use the result from subproblem a) to compute

$$\sum_{n=1}^{\infty}\frac{1}{n^4}.$$

   You may use without a proof that

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}.$$

   5 p

3. Heat transfer in a cylindrical shell is described by cylindrical coordinates $-\pi < \phi \le \pi$ and $0 < z < 1$ $(\rho = 1)$ with the equation

$$u_t = k(u_{zz} + u_{\phi\phi}).$$

   At equilibrium we have $u_t = 0$ and the equilibrium temperature satisfies Laplace's equation

$$u_{zz} + u_{\phi\phi} = 0$$

   in the shell. We assume that the temperature at $z = 0$ is kept constant at 0 and the temperature at $z = 1$ is kept constant to ϕ^4.

   Use separation of variables to determine the equilibrium temperature in the shell. *Hint: One of the functions involved satisfies a boundary value problem with periodic boundary conditions. The problem formulation from Problem 2 may be of use.* 20 *p*

4. Assume that $0 < \delta < \pi$ and let

$$f(x) = \begin{cases} 1 & \text{for } |x| \le \delta, \\ 0 & \text{for } \delta < |x| \le \pi, \end{cases}$$

and $f(x + 2\pi) = f(x)$ for every $x \in \mathbb{R}$.

a) Compute the Fourier coefficients of f on $(-\pi, \pi)$. 5 p

b) Show by using the result of subproblem a) that

$$\sum_{n=1}^{\infty} \frac{\sin(n\delta)}{n} = \frac{\pi - \delta}{2}$$

when $0 < \delta < \pi$. 5 p

c) Use Parseval's identity to show that

$$\sum_{n=1}^{\infty} \frac{\sin^2(n\delta)}{n^2\delta} = \frac{\pi - \delta}{2}.$$

5 p

d) What do you get if $\delta = \pi/2$ in subproblem c)? 5 p

5. a) Determine the eigenvalues and eigenfunctions of the Sturm–Liouville problem

$$xX'' + \frac{1}{2}X' + \lambda X = 0,$$
$$X(0) = X'(1) = 0.$$

Hint: $x = t^2$. 10 p

b) Write down the orthogonality property of the above eigenfunctions and verify it explicitly. 5 p

6. a) Represent the function

$$f(x) = \begin{cases} 1 - x & \text{då } 0 < x < 1, \\ 0 & \text{då } x \ge 1 \end{cases}$$

as a Fourier sine integral. 10 p

b) Compute

$$\int_0^{\infty} \frac{\alpha - \sin(\alpha)}{\alpha^2} \sin\left(\frac{\alpha}{2}\right) d\alpha.$$

5 p

The marked exams can be collected from Reine Elfsö in room 208, house 6. Email me at `maad@math.su.se` *and I will reply with your result and the date when the exams are ready to be collected.*