Homework 1 of 4

Logic, Stockholm University, Autumn 2014

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http://kurser.math.su.se/course/view.php?id=186

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Due Thursday 18 September, in class (or by email before class). Problems are marked with the per milles they count for on the final grade. This homework contains 4 problems.

1. (8‰) Either prove, or give a counterexample to, each of the following logical consequence statements:
   (a) $\vDash (P_1 \to \neg P_1)$
   (b) $(P_1 \wedge (P_2 \vee P_3)) \vDash (P_1 \wedge P_2) \vee (P_1 \wedge P_3)$
   (c) $(P_1 \vee (P_2 \wedge P_3)) \vDash (P_1 \vee P_2) \wedge (P_1 \vee P_3)$

2. (7‰) Give natural deduction proofs showing each of the following:
   (a) $(P_1 \vee P_2), \neg P_2 \vdash P_1$.
   (b) For any formula φ, $\varphi \vdash \top$.
   (c) $\vdash ((P_1 \to P_2) \to P_1) \to P_1$. (Hint: this requires *reductio ad absurdum*.)

3. (4‰) Give the $\vee$-introduction-1 and $\perp$-elimination cases of the proof of soundness.

4. (6‰) Prove, or give a counterexample to, the following statements:
   (a) For any set of formulas Γ and formulas ψ_1, ψ_2, if $\Gamma \vDash \psi_1 \wedge \psi_2$, then $\Gamma \vDash \psi_1$ and $\Gamma \vDash \psi_2$.
   (b) For any set of formulas Γ and formulas ψ_1, ψ_2, if $\Gamma \vDash \psi_1 \vee \psi_2$, then either $\Gamma \vDash \psi_1$ or $\Gamma \vDash \psi_2$.