Homework 2 of3

Logic, Stockholm University, Autumn 2014

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http://kurser.math.su.se/course/view.php?id=186

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Due Monday 13 October, in class (or by email before class). Problems are marked with the per milles they count for on the final grade. This homework contains 5 problems, and is worth 30‰ of the final grade.

N.B. There will only be 3 homeworks, not 4 as originally planned.

1. (6‰)
   (a) Show there is no deduction of $\bot$ from $\neg(\neg P_1)$.
   (b) Show there is no RAA-free deduction of $(\neg(\neg P_1)) \to P_1$. (Hint: look for a normal deduction.)

2. (5‰)
   (a) Explain why the following deduction of $P_1 \wedge P_2 \vdash P_1 \wedge P_1$ is not normal:

$$\dfrac{\dfrac{\dfrac{[P_1] \quad [P_1]}{P_1 \wedge P_1}\wedge\mathrm{I}}{P_1 \to P_1 \wedge P_1}\to\mathrm{I} \qquad \dfrac{P_1 \wedge P_2}{P_1}\wedge\mathrm{E}}{P_1 \wedge P_1}\to\mathrm{E}$$

   (b) Give a normal deduction of $P_1 \wedge P_2 \vdash P_1 \wedge P_1$.

3. (5‰) Which of the following assertions are correct, and if not, why? (Take the arity type to be $\langle 1, 2; 2\rangle$.)
   (a) $P_1(f_1(x_1), f_1(x_2)) \in \mathrm{Form}[x_1, x_2]$
   (b) $\exists x_3(x_3 \doteq x_1) \in \mathrm{Form}[x_1]$
   (c) $(x_2 \doteq x_3) \to (x_3 \doteq x_2) \in \mathrm{Form}(\emptyset)$
   (d) $P_1(f_2(x_5, x_5), x_5) \in \mathrm{Form}[x_5, x_6, x_7, x_8]$
   (e) $\forall x_1\big((\exists x_2(P_1(x_2, x_3))) \wedge (f_1(x_1) \doteq x_2)\big) \in \mathrm{Form}[x_3]$

4. (8‰) In each of the following structures, find the interpretation

$$[\![\, x_1, x_2 \mid P_1(x_1, x_2) \to P_1(x_2, x_1) \,]\!].$$

   (a) $\langle\, \mathbb{Z};\ ;\ = \,\rangle$

   (b) $\langle\, \mathbb{Z};\ ;\ < \,\rangle$

   (c) $\langle\, \mathbb{Z};\ ;\ \leq \,\rangle$

   (d) $\langle\, \mathbb{Z};\ ;\ \{(m,n) \in \mathbb{Z}^2 \mid m \text{ divides } n\} \,\rangle$

   (For $m, n \in \mathbb{Z}$, "m divides n" means "there is some integer k such that $n = km$".)

5. (6‰) Find a structure in which the closed formula $\forall x_1(\exists x_2(x_1 \doteq f_1(x_2, x_2)))$, and another structure in which it does not hold.