Homework set 3

HW1. Let $E \to B$ be a principal fibration with fiber $K(G,n)$, and $A \to X$ a cofibration. In class we showed how to associate to the diagram

$$\begin{array}{ccc} A & \longrightarrow & E \\ \downarrow & & \downarrow \\ X & \longrightarrow & B \end{array}$$

a natural *obstruction class* $\omega \in H^{n+1}(X,A;G)$ which vanishes precisely if a lift exists in the diagram. Show that given two lifts $f_1, f_2 : X \to E$, there is a natural class $\omega' \in H^n(X,A;G)$ which vanishes precisely if $f_1 \simeq f_2$ as lifts; similarly, if H_1 and H_2 are homotopies between f_1 and f_2 then there is a further obstruction class $\omega'' \in H^{n-1}(X,A;G)$ which vanishes precisely if H_1 and H_2 are themselves homotopic rel A over B, et cetera.

HW2. Consider the Postnikov tower of S^2. Show that the k-invariant

$$k_2 \in H^4(K(\pi_2(S^2),2),\pi_3(S^2)) = H^4(\mathbb{CP}^\infty,\mathbb{Z})$$

is a generator for this abelian group.

Deadline: 2022–11–01. If you have used any resources outside the course literature/lecture notes, please indicate this in your solution. Similarly if you have discussed the problems with another student. Hand in your solutions by e-mail to: dan.petersen@math.su.se