

HOMEWORK SET 5

Consider a principal S^3 -bundle

$$S^3 \rightarrow M \rightarrow S^2 \times S^2$$

with non-trivial Euler class $e \in H^4(S^2 \times S^2; \mathbb{Q})$.

- (1) Compute the rational homotopy groups and the rational cohomology ring of M .
- (2) Construct a minimal model for M . (Hint: Use a rational Postnikov tower for M .)
- (3) Show that the minimal model for M is different from the minimal model for $H^*(M; \mathbb{Q})$. (Hint: Compute the latter in low degrees using the algebraic algorithm.)

Comments: If X and $H^*(X; \mathbb{Q})$ have isomorphic minimal models, then X is called *formal*.

Compact Kähler manifolds (e.g. smooth complex projective algebraic varieties) are always formal by a theorem of Deligne–Griffiths–Morgan–Sullivan.

Simply connected closed manifolds of dimension ≤ 6 are always formal by a theorem of Miller.

The above provides an example of a non-formal simply connected closed 7-manifold.

Deadline: 2022–11–29. If you have used any resources outside the course literature/lecture notes, please indicate this in your solution. Similarly if you have discussed the problems with another student. Hand in your solutions by e-mail to: alexb@math.su.se