

HOMEWORK SET 6

Let  $U(n)$  be the group of unitary  $n \times n$ -matrices.

- (1) Compute the rational homotopy groups of  $U(n)$ .  
(Hint: construct, or look up the construction of, a fibration sequence

$$U(n-1) \rightarrow U(n) \rightarrow S^{2n-1}.)$$

- (2) Suppose that  $X$  is a simply connected space such that  $\pi_*(X) \otimes \mathbb{Q}$  is finite dimensional and concentrated in even degrees. Show that  $H^*(X; \mathbb{Q})$  is isomorphic to a polynomial ring on  $\text{Hom}(\pi_*(X), \mathbb{Q})$ .
- (3) Compute the rational cohomology ring of  $BU(n)$  using (1) and (2).  
(Recall: if  $G$  is a topological group, then  $\Omega BG \simeq G$ .)

*Deadline: 2022-12-15. If you have used any resources outside the course literature/lecture notes, please indicate this in your solution. Similarly if you have discussed the problems with another student. Hand in your solutions by e-mail to: alexb@math.su.se*