MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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Tentamensskrivning i
Topologi, MM7052
7.5 hp
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(1) Let $\mathbb{Z}$ denote the set of integers. Declare a subset $U \subseteq \mathbb{Z}$ to be open if

$$n \in U \Leftrightarrow -n \in U$$

for all $n \in \mathbb{Z}$.

(a) Show that this defines a topology on $\mathbb{Z}$. (2p)

(b) Is $\mathbb{Z}$ Hausdorff in this topology? Compact? Second countable? Motivate your answers. (3p)

(2) Let $p\colon X \to Y$ be a quotient map and let $A \subseteq X$ be a subspace.

(a) Show that if A is open in X and if p is an open map, then the restriction $p|_A\colon A \to p(A)$ is a quotient map. (3p)

(b) Give an example to show that $p|_A\colon A \to p(A)$ is not a quotient map in general. (2p)

(3) A topological space X is called *totally disconnected* if the only connected subsets are the singletons $\{x\}$ for $x \in X$.

(a) Prove that the real line $\mathbb{R}$ with the topology defined by the basis $(a, b]$ for $a < b$ is totally disconnected. (2p)

(b) Consider the space $C = \{0, 1\}^{\mathbb{Z}}$ of functions $f\colon \mathbb{Z} \to \{0, 1\}$ equipped with the coarsest topology for which all evaluation maps

$$ev_n\colon \{0,1\}^{\mathbb{Z}} \to \{0,1\}, \quad ev_n(f) = f(n),$$

are continuous, where $\{0, 1\}$ is given the discrete topology. Show that C is totally disconnected but not discrete. (3p)

(4) There is an action of the cyclic group C_2 of order 2 on the torus $T = S^1 \times S^1$ defined by letting the non-trivial element act by

$$(x, y) \mapsto (y, x).$$

Show that the orbit space T/C_2 is homeomorphic to the Möbius band. (5p)
(Hint: You may find it useful to argue using polygonal presentations.)

(5) Show that every map $\mathbb{RP}^n \to S^1$ is homotopic to a constant map if $n \geq 2$. (5p)
(Hint: Use covering space theory.)

(6) Compute the fundamental group of the complement of the three coordinate axes in $\mathbb{R}^3$. (5p)