Abstract

There are two major approaches for using topoi for the interpretation of quantum mechanics, the contravariant and the covariant approach pioneered by Isham–Butterfield [1–4] and Landsman–Spitters–Heunen [5] respectively. To each topoi, there is an internal language in which constructive logic can be interpreted. This internal language is heavily used in the covariant approach, whereas any use in the contravariant approach is much more hidden.

There is also a related approach, used to study more general forms of measurement scenarios as sheaves, pioneered by Abramsky–Brandenburger [6]. This approach is general enough to make the contravariant models a special case.

In this thesis, we develop a method for considering (a variation of) these general measurement scenarios in a covariant setting using some of the tools developed by Landsman–Spitters–Heunen. The measurement scenarios are first presented as contravariant functors on a poset. We then topologize the poset using the Alexandrov topology and look at the presheaf as a bundle. This gives us a locale in the category of covariant presheaves.

We show that the locales constructed in this way are always internally spatial. We also use the constructive Gelfand duality described by Banaschewski–Mulvey [7] to get an internal commutative C^*-algebra corresponding to this locale. Lastly, we look at how to construct a state as defined by Landsmann–Spitters–Heunen.

We also consider two examples thoroughly, Spekken's Toy Model [8] and the Popescu–Rohrlich-box [9]. In doing this, we construct a new proof that the Popescu–Rohrlich-box cannot be modelled by quantum theory, without using the Tsirelson's bound.