Abstract

Many naturally occurring processes follow some principle of optimality, e.g. the principle of minimisation of potential energy. As a consequence, optimisation provides invaluable tools in physics and engineering. Here, models for fracture of materials are investigated using calculus of variations and optimal control theory.

Three different models for predicting when (or if) fracture occurs are presented and investigated: Griffith's model, the variational approach to fracture and a set of models referred to as gradient regularised fracture models. They are all based on the principle of minimisation of potential energy, given an extended notion of potential energy as consisting of the sum of the elastic energy and the surface energy of the (eventual) fracture surface.

The specific problem under consideration is that of uniaxial fracture of a rod subjected to a prescribed displacement in one end while the other end remains fixed. It is investigated in terms of solutions to the fracture problem given by the three different models, with emphasis on the gradient regularised models. Necessary conditions for the existence of minimisers are derived using both Euler-Lagrange equations and Pontryagin's minimum principle. Sufficient conditions for the existence of minimisers are investigated using the second variation. The essay also includes an introduction to calculus of variations and the related topic of optimal control theory.