MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sofia Tirabassi

Tentamensskrivning i
Mathematics Methods for Economist MM3001
2023-03-13

Instructions:

- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- You can use the formula sheet that come with the exam.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 3 Taylor polynomial of the function

$$f(x) = (x-1)\ln(x^2+1),$$

around the point $x_0 = 0$, and use it to give an approximation of $f(0.1)$.

(2) Geometric Series: Consider the following sequence:

$$a_0 = 3, \quad a_1 = \frac{3}{(1+2p)^2}, \quad a_2 = \frac{3}{(1+2p)^4}, \quad a_3 = \frac{3}{(1+2p)^6}, \ldots$$

(a) (2 pt) Show that a_n determines a geometric progression, compute the geometric ratio and give an expression for

$$\sum_{n=2}^{6} a_n$$

(b) (2pt) Determine for which value of p the infinite series

$$S(p) = \sum_{n=0}^{\infty} 3(1+2p)^{-2n}$$

converges.

(c) (1pt) Determine if there is a p such that $S(p) = \frac{7}{4}$.

(3) Consider the function $f(x) = \frac{x^2-9}{x^2-4}$.

(a) (1pt) Find the natural domain of $f(x)$ and the solutions to $f(x) = 0$.
(b) (2pt) Find where the function is increasing or decreasing. Find the critical points of $f(x)$ and determine their type.
(c) (1pt) Find the max and min value of the function on the interval $[-1, 1]$.
(d) (1pt) Compute $\lim_{x\to\pm\infty} f(x)$ and sketch the graph of f.

(4) Compute the following integrals:

(a) (2.5 pt) $\int \left(\frac{\ln(y)}{((\ln(y))^2+1)} \frac{1}{y} + \sqrt[3]{y^8} \right) dy$,

(b) (2.5pt) $\int_0^1 xe^{x+1} dx$.

(5) Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 4+c \\ -4 & 1 & 0 \\ c & 0 & -2 \end{pmatrix}$$

(a) (2 pt) Compute the determinant of A, $|A|$ as a function of c.

(b) (1 pt) Find all the values of c for which A is not invertible.

(c) (2 pt) Find how many solutions has the following linear system:

$$\begin{cases} 2x & -y & +5z & = & 4 \\ -4x & +y & & = & 13 \\ x & & -2z & = & -8 \\ 5x & -y & -2z & = & -21 \end{cases}$$

(6) Consider the two variables function

$$f(x,y) = y^3 + 2x^2 + 4x - 27y + 100$$

defined on the square

$$D = \{(x,y) \mid -3 \leq x \leq 0,\ -3 \leq y \leq 0\}$$

(a) (2pt) Find all the critical points of $f(x,y)$ - even those lying outside D and determine their type.

(b) (2pt) Determine the maximum and minimum points of f on the *boundary* of D. (In order to get credit you have to explain what you are doing, the correct answer without the right explanation will not be accepted)

(c) (1 pt) Determine the minimum and the maximum value of $f(x,y)$ on D. (In order to get credit you have to explain what you are doing, the correct answer without the right explanation will not be accepted)

GOOD LUCK!!!

Senska texten, (formular finns ovanför)

(1) (5pt) Beräkna grad 3 Taylor polinom till funktioner
$$f(x) = (x-1)\ln(x^2+1),$$
omkring punkten $x_0 = 0$, och använda det för approximera $f(0.1)$.

(2) Geometriska Serier: Betrakta följande talföjld
$$a_0 = 3, \quad a_1 = \frac{3}{(1+2p)^2}, \quad a_2 = \frac{3}{(1+2p)^4}, \quad a_3 = \frac{a}{(1+2p)^6}, \ldots$$
(a) (2pt) visa att talföjlden är geometrisk och ge en formel för att räkna
$$\sum_{n=2}^{6} a_n$$
(b) (2 pt) Bestämm för vilka p den oendliga series nedanfór convergerar:
$$S(p) = \sum_{n=0}^{\infty} 3(1+2p)^{-2n}$$
(c) (1pt) Bestäm om det finns p sådan att $S(p) = \frac{7}{4}$.

(3) Betrakta funktionen $f(x) = \frac{x^2-9}{x^2-4}$.
(a) (1pt) Hitta var funktionen är definerad ock lösningar till $f(x) = 0$
(b) (2pt) Hitta alla de kritiska punkterna och bestäm dess typ. Hitta var funktioner är växande eller avtagende.
(c) (1pt) Bestämm den max och min värde till funktionen på intervvallen $[-1, 1]$.
(d) (1pt) Räkna $\lim_{x\to\pm\infty} f(x)$ och skissa grafen till f.

(4) Räkna de följande integralerna:
(a) (2.5 pt) $\displaystyle\int \left(\frac{\ln(y)}{((\ln(y))^2+1)} \frac{1}{y} + \sqrt[3]{y^8} \right) dy,$
(b) (2.5pt) $\displaystyle\int_0^1 x e^{x+1} dx.$

(5) Betrakta matrisen
$$A = \begin{pmatrix} 2 & -1 & 4+c \\ -4 & 1 & 0 \\ c & 0 & -2 \end{pmatrix}$$
(a) (2 pt) Räkna determinanter till A, $|A|$ som en funktion av c.
(b) (1 pt) Hitta alla värder c sådan att A inte är invertebär.
(c) (2 pt) Hitta hur många lösningar har systemet
$$\begin{cases} 2x & -y & +5z & = & 4 \\ -4x & +y & & = & 13 \\ x & & -2z & = & -8 \\ 5x & -y & -2z & = & -21 \end{cases}$$

(6) Betrakta den foljande funktionen av två variabler
$$f(x, y) = y^3 + 2x^2 + 4x - 27y + 100$$
som defineras i fyrkanten
$$D = \{(x, y) \mid -3 \le x \le 0, \; -3 \le y \le 0\}$$

(a) (2pt) Hitta alla kritiska punkter till $f(x, y)$ - punkter som ligger utanför D också behövs att hitta.
(b) (2pt) Hitta den storsta och den minsta punkter till f *gränsen* av D .
(c) (1 pt) Beräkna den storsta och den minst värden till f på D.

LYCKA TILL!!!