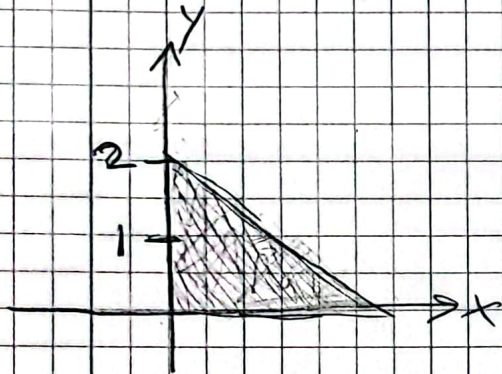


6.11 Beräkna

$$\iint_D e^{-2x-3y} dx dy$$

D: $x \geq 0, y \geq 0, 2x + 3y \leq 6$

Rita upp området: $2x - 6 \leq -3y$
 $-\frac{2}{3}x + 2 \leq y$



Gränser: $0 \leq y \leq 2$
 $0 \leq x \leq 3$

$$-\frac{2}{3}x + 2 = 0$$

$$-\frac{2}{3}x = -2$$

$$\frac{1}{3}x = 1$$

$$x = 3$$

$$\int_0^2 \left(\int_0^3 e^{-2x-3y} dx \right) dy$$

$$\int_0^2 e^{-2x-3y} dx = \left[-\frac{e^{-2x-3y}}{2} \right]_{x=0}^{x=3} = -\frac{e^{-6-3y}}{2} + \frac{e^{-3y}}{2}$$

$$= \frac{e^{-3y}}{2} - \frac{e^{-3y}}{2} \cdot e^{-6} = \frac{e^{-3y}}{2} (1 - e^{-6})$$

$$\int_0^2 \frac{e^{-3y}}{2} (1 - e^{-6}) dy = \left(\frac{1 - e^{-6}}{2} \right) \int_0^2 e^{-3y} dy$$

$$= \frac{1 - e^{-6}}{2} \left[-\frac{e^{-3y}}{3} \right]_{y=0}^{y=2} = \frac{1 - e^{-6}}{2} \left[-\frac{e^{-6}}{3} + \frac{e^0}{3} \right]$$

$$= \frac{1 - e^{-6}}{2} \left(\frac{1}{3} - \frac{e^{-6}}{3} \right) = \frac{1 - e^{-6}}{6} (1 - e^{-6})$$

$$= (6 - 6e^{-6}) (1 - e^{-6}) = 6 - 6e^{-6} - 6e^{-6} + 6e^{-12} = 6 - 12e^{-6} + 6e^{-12}$$