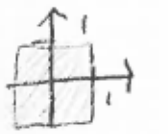
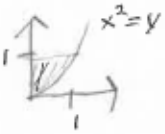
6.5 $\Delta = \{(x,y) : |x| \le 1,\ |y| \le 1\}$ 

$$\int x e^{-(x^2+y^2)}\,dx = \left[\begin{matrix} x^2 = t \\ 2x\,dx = dt \end{matrix}\right] = \int e^{-(t+y^2)}\frac{dt}{2} = \frac{1}{2}e^{-(t+y^2)} + C$$

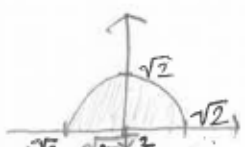
$$\iint_\Delta x e^{-(x^2+y^2)}\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1} x e^{-(x^2+y^2)}\,dx\,dy = \int_{-1}^{1}\left[\frac{-1}{2}e^{-(x^2+y^2)}\right]_{-1}^{1} dy$$

$$= \int_{-1}^{1}\left(\frac{-1}{2}e^{-(1+y^2)} - \frac{-1}{2}e^{-(1+y^2)}\right)dy = \int_{-1}^{1} 0\,dy = 0$$

6.12 $D : x^2 \le y \le 1,\ x \ge 0$ 

$$\iint_D \frac{x}{1+y^2}\,dx\,dy = \int_0^1\int_0^{\sqrt{y}} \frac{x}{1+y^2}\,dx\,dy = \int_0^1 \left[\frac{x^2}{2}\,\frac{1}{1+y^2}\right]_0^{\sqrt{y}} dy = \int_0^1 \frac{y}{2}\,\frac{1}{1+y^2}\,dy$$

$$= \left[\begin{matrix} y^2 = t \\ 2y\,dy = t \end{matrix}\right] = \int_0^1 \frac{1}{2}\,\frac{1}{1+t}\,\frac{dt}{2} = \left[\frac{1}{4}\ln(1+t)\right]_0^1 = \frac{1}{4}\ln(2)$$

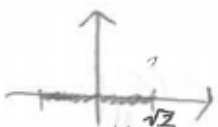
B6 a) $D_a : x^2 + y^2 \le 2,\ y > 0$ 

$$\iint_{D_a} (x^2 y + y\sin x^9)\,dx\,dy = \int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2-x^2} (x^2 y + y\sin x^9)\,dy\,dx = \int_{-\sqrt{2}}^{\sqrt{2}}\left[x^2\frac{y^2}{2} + \frac{y^2}{2}\sin x^9\right]_0^{2-x^2} dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}}\left(x^2\,\frac{2-x^2}{2} + \underbrace{\frac{2-x^2}{2}}_{\text{gerade}}\,\underbrace{\sin x^9}_{\text{ungerade}}\right)dx = \int_{-\sqrt{2}}^{\sqrt{2}}\left(x^2 - \frac{x^4}{2}\right)dx$$

$$= \left[\frac{x^3}{3} - \frac{x^5}{10}\right]_{-\sqrt{2}}^{\sqrt{2}} = \ldots = \frac{8\sqrt{2}}{15}$$

b) $D_b : x^2 + y^2 \le 2,\ y \ge 0$

$D_b \setminus D_a = [-\sqrt{2}, \sqrt{2}] \times \{0\}$ hallmingd 

si: $\iint_{D_b} (x^2 y + y\sin x^9)\,dx\,dy = \iint_{D_a} (x^2 y + y\sin x^9)\,dx\,dy = \frac{8\sqrt{2}}{15}$

B8 $$\int_{\frac{1}{2}}^{1}\left(\int_{\frac{\pi}{v}}^{2\pi} \sin(xv)\,dx\right)dv + \int_1^2\left(\int_{\frac{\pi}{y}}^{2\pi} \sin(xy)\,dx\right)dy = \left[\begin{matrix} 2v = y \\ 2\,dv = dy \end{matrix}\right]$$

$$= \int_1^2\left(\int_{\frac{\pi}{y/2}}^{2\pi} \sin\left(x\,\frac{y}{2}\right)dx\right)\frac{dy}{2} + \int_1^2\left(\int_{\frac{\pi}{y}}^{2\pi} \sin(xy)\,dx\right)dy$$

$$= \int_1^2\left(\left[\frac{-2}{y}\cos\left(x\,\frac{y}{2}\right)\right]_{\frac{2\pi}{y}}^{2\pi}\frac{1}{2} + \left[-\frac{1}{y}\cos(xy)\right]_{\frac{\pi}{y}}^{\frac{2\pi}{y}}\right)dy = \int_1^2\left(\frac{-1}{y}\cos(\pi y) + \frac{1}{y}\cos(\pi) - \frac{1}{y}\cos(2\pi) + \frac{1}{y}\cos(\pi y)\right)$$

$$= \int_1^2 \frac{-2}{y}\,dy = \left[-2\ln(y)\right]_1^2 = -2\ln(2)$$