9.8 C: enhetscirkeln positivt orienterad, vill beräkna $\oint_C (e^{\sin x}-x^2y)dx + e^{y^2}dy$

$$\oint_C (e^{\sin x}-x^2 y)\,dx + e^{y^2}dy = \iint_{x^2+y^2\le 1} \left(\frac{\partial}{\partial x}(e^{y^2}) - \frac{\partial}{\partial y}(e^{\sin(x)}-x^2y)\right)dx\,dy$$

$$= \iint_{x^2+y^2\le 1} x^2\, dx\,dy = \int_0^1\int_{-\pi}^{\pi} r^2\cos^2(\theta)\, r\, d\theta\, dr = \int_0^1 r^3\, dr \int_{-\pi}^{\pi}\cos^2(\theta)\,d\theta$$

$$= \left[\frac{r^4}{4}\right]_0^1 \int_{-\pi}^{\pi} \frac{\cos(2\theta)+1}{2}\,d\theta = \frac14\left[\frac{\sin(2\theta)}{4}+\frac{\theta}{2}\right]_{-\pi}^{\pi} = \frac14\pi$$

9.10 γ cirkel $(x-a)^2+(y-b)^2=r^2$ ett varv moturs
vill beräkna $\oint_\gamma y^2dx+x^2dy$

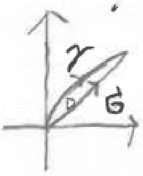
$$\oint_\gamma y^2dx + x^2dy = \iint_{(x-a)^2+(y-b)^2\le r^2}(2x-2y)\,dx\,dy = \begin{bmatrix} x-a=\rho\cos(\theta) \\ y-b=\rho\sin(\theta)\end{bmatrix}$$

$$=\int_0^r\int_{-\pi}^{\pi}\big(2(\rho\cos(\theta)+a)-2(\rho\sin(\theta)+b)\big)\rho\, d\theta\, d\rho$$

$$=\int_0^r \Big[2(\rho\sin(\theta)+\rho\cos(\theta)+a\theta-b\theta)\rho\Big]_{-\pi}^{\pi} d\rho$$

$$=\int_0^r 4\pi(a-b)\rho\, d\rho = \left[2\pi(a-b)\rho^2\right]_0^r = 2\pi(a-b)r^2$$

356 γ kurva $y=\sqrt{x}$ $\quad 0\le x\le 1$, $\bar F=(\sin(y-x),\ 2xy+\sin(x-y))$
Vill beräkna $\int_\gamma \bar F\cdot d\bar r$



vi definierar σ som linjen $(0,0)$ till $(1,1)$ $(\bar r(t)=(t,t)\quad t\in[0,1])$
och D som området mellan γ och σ dvs $0\le x\le 1,\ x\le y\le\sqrt{x}$

$$\int_\sigma \bar F\cdot d\bar r - \int_\gamma \bar F\cdot d\bar r = \iint_D\left(\frac{\partial}{\partial x}(2xy+\sin(x-y)) - \frac{\partial}{\partial y}(\sin(y-x))\right)dx\,dy$$

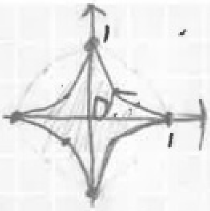
$$=\int_0^1\int_x^{\sqrt x}\big(2y+\cos(x-y)-\cos(y-x)\big)\,dy\,dx$$

$$=\int_0^1\left[y^2\right]_x^{\sqrt x}dx = \int_0^1(x-x^2)\,dx = \frac12-\frac13=\frac16$$

$$\Rightarrow \int_\gamma \bar F\cdot d\bar r = \int_\sigma \bar F\cdot d\bar r - \frac16 = \int_0^1(\sin(t-t),\ 2t\cdot t+\sin(t-t))\cdot(dt,dt) - \frac16$$

$$=\int_0^1 2t^2\,dt - \frac16 = \frac23-\frac16=\frac12$$

25 $\gamma: \begin{cases} x = \cos^3 t \\ y = \sin^3(t) \end{cases} \quad 0 \le t \le 2\pi$



$\bar{r}(t) = (x(t), y(t))$

$\bar{r}(0) = (1, 0)$

$\bar{r}(\frac{\pi}{4}) = (\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}})$

$\bar{r}(\frac{\pi}{2}) = (0, 1)$

$\vdots$

Vill skissa γ, och beräkna arean av område D som begränsas av γ

$$\mu(D) = \iint_D dx\,dy = \iint_D \frac{1}{2}\left(\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y)\right) dx\,dy = \frac{1}{2}\int_\gamma -y\,dx + x\,dy$$

$$= \frac{1}{2}\int_0^{2\pi} -\sin^3(t)\,(3\cos^2(t)(-\sin(t))\,dt) + \cos^3(t)\,(3\sin^2(t)\cos(t)\,dt)$$

$$= \frac{1}{2}\int_0^{2\pi} 3\cos^2(t)\sin^2(t)\,(\sin^2(t) + \cos^2(t))\,dt$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{3}{4}\sin^2(2t)\,dt = \frac{1}{2}\int_0^{2\pi} \frac{3}{4}\,\frac{1-\cos(4t)}{2}\,dt$$

$$= \int_0^{2\pi} \frac{3}{16}(1 - \cos(4t))\,dt = \left[\frac{3}{16}\left(t - \frac{1}{4}\sin(4t)\right)\right]_0^{2\pi} = \frac{3}{8}\pi$$