10.3 $\bar{F} = (y, (x+z)^2, (x-z)^2)$, γ ges av $y=x^2$, $z=0$ från $(0,0,0)$ till $(2,4,0)$

Beräkna $\int_\gamma \bar{F}\cdot d\bar{r}$

$\bar{r}(t) = (t, t^2, 0)$ $\quad t\in[0,2]$

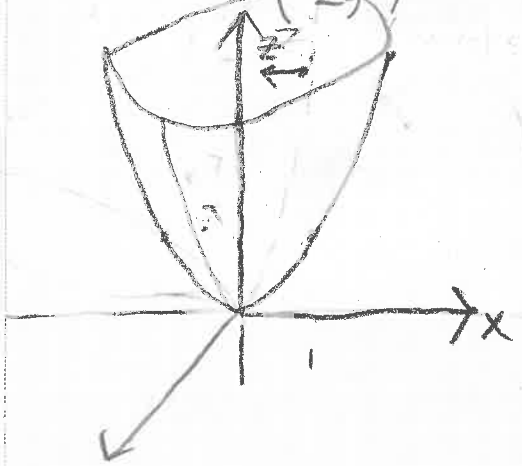
$$\int_\gamma \bar{F}\cdot dr = \int_0^2 (t^2, (t+0)^2, (t-0)^2)\cdot(1, 2t, 0)\,dt$$

$$= \int_0^2 (t^2+2t^3)\,dt = \left[\frac{t^3}{3} + \frac{t^4}{2}\right]_0^2 = \frac{8}{3} + 8 = \frac{32}{3}$$

378 Bestäm normalvektorn till ytan $\bar{r}(s,t) = (s\cos(t), s\sin(t), s^2)$, $s\geq 0$, $t\in[0,2\pi]$ och tolka grafiskt

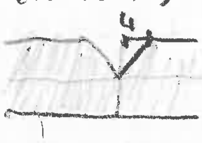
$$\bar{r}'_s = \begin{pmatrix}\cos(t)\\ \sin(t)\\ 2s\end{pmatrix} \quad \bar{r}'_t = \begin{pmatrix}-s\sin(t)\\ s\cos(t)\\ 0\end{pmatrix} \quad \bar{r}'_s\times\bar{r}'_t = \begin{pmatrix}\sin(t)\cdot 0 - 2s(s\cos(t))\\ (-s\sin(t))\,2s - 0\cdot\cos(t)\\ \cos(t)\,s\cos(t) - \sin(t)(-s\sin(t))\end{pmatrix}$$

$$= \begin{pmatrix}-2s^2\cos(t)\\ -2s^2\sin(t)\\ s\end{pmatrix}$$

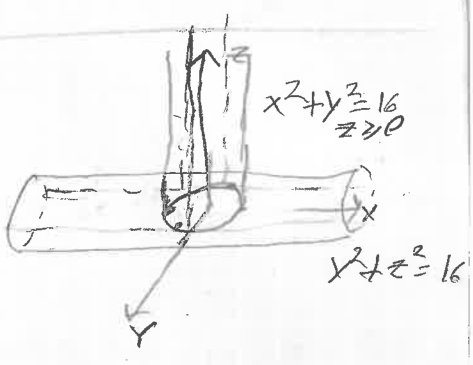


Normalvektorn pekar in och upp mot z-axeln

8.18 I cylinder med radie 4cm stansas ett "cirkulärt" hål enligt figur



Vad är arean av hålet?



cylinder ges av $y^2+z^2 = 4^2 = 16$

stansar ut $x^2+y^2 \leq 16$, $z\geq 0$

$$\Leftrightarrow \begin{cases} y^2+z^2=16\\ x^2-z^2\leq 0\\ z\geq 0\end{cases} \Leftrightarrow \begin{cases} y^2+z^2=16\\ |x|\leq|z|=z\\ z\geq 0\end{cases}$$

$\bar{r}(s,t) = (s, 4\cos(t), 4\sin(t))$ $\quad 0\leq t\leq\pi$, $-4\sin(t)\leq x\leq 4\sin(t)$

$\bar{r}'_s = (1,0,0)$ $\quad \bar{r}'_t = (0, -4\sin(t), 4\cos(t))$ $\quad \bar{r}'_s\times\bar{r}'_t = (0, -4\cos(t), -4\sin(t))$

$|\bar{r}'_s\times\bar{r}'_t| = \sqrt{16\cos^2(t)+16\sin^2(t)} = 4$

$$\iint_Y dS = \int_0^\pi\int_{-4\sin(t)}^{4\sin(t)} 4\,ds\,dt = \int_0^\pi 32\sin(t)\,dt = [-32\cos(t)]_0^\pi = 64$$

Så area 64 cm^2