10.53 γ: $y^2+z^2=1$ $x=1$ genomlöps negativ sedd från origo

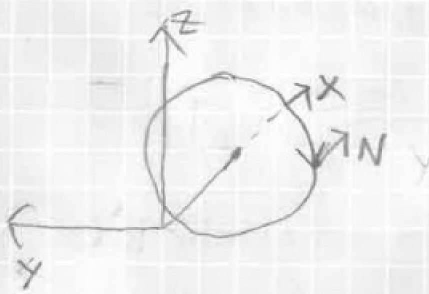
Beräkna arbetet $u=(xyz,\ xy^2z^3-z,\ xy^3z^2)$ uträttar

Låt Y; $y^2+z^2\leq 1, x=1$ $\bar{N}=(1,0,0)$

$$\operatorname{rot}(u)=\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}\times\begin{pmatrix}xyz\\ xy^2z^3-z\\ xy^3z^2\end{pmatrix}=\begin{pmatrix}3xy^2z^2-(3xy^2z^2-1)\\ y-y^3z^2\\ y^2z^3-z\end{pmatrix}$$

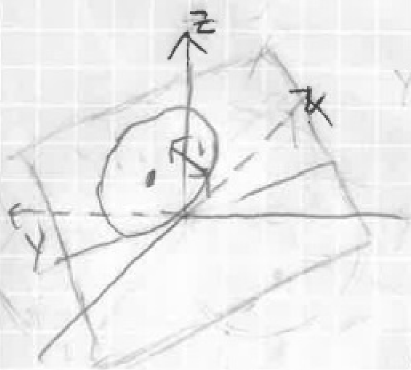
$$=\begin{pmatrix}1\\ y-y^3z^2\\ y^2z^3-z\end{pmatrix}$$

$$\int_\gamma u\cdot dr=\iint_Y \operatorname{rot}(u)\cdot\bar{N}\,dS$$
$$=\iint_Y (1, y-y^3z^2, y^2z^3-z)\cdot(1,0,0)\,dS$$
$$=\iint_Y dS=\pi$$



B103 Beräkna $\int_\gamma F\cdot dr$ där $F=(4x^3y^3+z,\ 3x^4y^2+z^2,\ z^5)$
γ: $(x-1)^2+(y-2)^2=4$, $z=x+2y$ orienterad moturs uppifrån sett

$$\operatorname{rot}F=\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}\times\begin{pmatrix}4x^3y^3+z\\ 3x^4y^2+z^2\\ z^5\end{pmatrix}=\begin{pmatrix}0-2z\\ 1-0\\ 12x^3y^2-12x^3y^2\end{pmatrix}=\begin{pmatrix}-2z\\ 1\\ 0\end{pmatrix}$$



Y: $(x-1)^2+(y-2)^2\leq 4, z=x+2y$

$$\bar{r}(x,y)=\begin{pmatrix}x\\ y\\ x+2y\end{pmatrix}$$

$$\bar{r}'_x\times\bar{r}'_y=\begin{pmatrix}1\\ 0\\ 1\end{pmatrix}\times\begin{pmatrix}0\\ 1\\ 2\end{pmatrix}=\begin{pmatrix}-1\\ -2\\ 1\end{pmatrix}$$

$$\int_\gamma F\,dr=\iint_Y \operatorname{rot}F\cdot N\,dS=\iint_{(x-1)^2+(y-2)^2\leq 4}(-2(x+2y),1,0)\cdot(-1,-2,1)$$

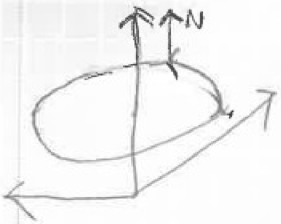
$$=\iint_{(x-1)^2+(y-2)^2\leq 4}(2x+4y-2)\,dx\,dy=\begin{bmatrix}x=r\cos(\theta)+1\\ y=r\sin(\theta)+2\end{bmatrix}$$

$$=\int_0^2\int_{-\pi}^{\pi}(2(r\cos(\theta)+1)+4(r\sin(\theta)+2)-2)\,r\,d\theta\,dr$$

$$=\int_0^2\left[2r\sin(\theta)-4r\cos(\theta)+8\theta\right]_{-\pi}^{\pi}r\,dr$$

$$=\int_0^2 16\pi r\,dr=8\pi\,2^2=32\pi$$

3105 Beräkna $\int_\gamma F\cdot dr$ där $F=(z\cos(y), -xz\sin(y)+2x, x\cos(y))$ och γ är skärningskurvan mellan ytan $x^2+y^2+z^2=4$ och planet $z=1$ moturs uppifrån



$$\text{rot } F = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} z\cos(y) \\ -xz\sin(y)+2x \\ x\cos(y) \end{pmatrix} = \begin{pmatrix} -x\sin(y)-(-x\sin(y)) \\ \cos(y)-\cos(y) \\ -z\sin(y)+2-(-z\sin(y)) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$D: x^2+y^2\leq 3,\ z=1$

$$\int_\gamma F\, dr = \iint_D \text{rot } F\cdot N\, dS = \iint_{x^2+y^2\leq 3} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} dx\,dy = 2\cdot 3\pi = 6\pi$$

10.40 Visa

a) $\nabla\times(\nabla f)=0$

$$\nabla\times\nabla f = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} f'_x \\ f'_y \\ f'_z \end{pmatrix} = \begin{pmatrix} f''_{zy}-f''_{yz} \\ f''_{xz}-f''_{zx} \\ f''_{yx}-f''_{xy} \end{pmatrix} = 0$$

b) $\nabla\cdot(\nabla\times U)=0$

$$U=(P,Q,R): \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}\cdot\left(\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}\times\begin{pmatrix} P \\ Q \\ R \end{pmatrix}\right) = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}\cdot\begin{pmatrix} R'_y-Q'_z \\ P'_z-R'_x \\ Q'_x-P'_y \end{pmatrix} = R''_{yx}-Q''_{zx}+P''_{zy}-R''_{xy}+Q''_{xz}-P''_{yz}$$

$=0$

c) $\nabla\cdot(\nabla f)=\Delta f$

$$\nabla\cdot(\nabla f) = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}\cdot\begin{pmatrix} f'_x \\ f'_y \\ f'_z \end{pmatrix} = f''_{xx}+f''_{yy}+f''_{zz}=\Delta f$$