STOCKHOLM UNIVERSITY
Department of Mathematics
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Examination in
Mathematics for Economic and Statistical Analysis
MM1005, Fall term; 7,5 ECTS
Tuesday 31 October, 2023

Instructions:
- During the exam you may not use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers.
- Use natural language, not just mathematical symbols. Write clearly and legibly
- Mark your final answer to each question clearly by putting a a box around it.
- **Do not write two exercises on the same sheet.**

Grades: Each solved problem is awarded by up to 10 points. At least 35 points would guarantee grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. (a) Find all the real values x satisfying the folllowing inequality

$$\begin{vmatrix} 1 & 1 & x \\ x & 1 & 1 \\ 1 & x & 1 \end{vmatrix} \geq \begin{vmatrix} 2x & -1 \\ -8 & x \end{vmatrix}.$$

   (b) Determine the value of the parameter b such that $\lim_{x\to 0} \frac{e^{bx}-x-1}{2x} = -3$.

Solution (a) $x \geq -2$.

(b) $b = -5$.

2. Calculate, argumenting your answer, the integrals

$$\text{a)} \int \left(e^{\sqrt{x}} + \frac{1}{x-1} \right) \mathrm{d}x \qquad \text{b)} \int_0^{+\infty} \frac{x^2+5}{(x^3+15x+8)^{4/3}} \mathrm{d}x$$

Solution

$$\int \left(e^{\sqrt{x}} + \frac{1}{x-1} \right) \mathrm{d}x = 2e^{\sqrt{x}} \left(\sqrt{x} - 1 \right) + \log(x-1) + C. \qquad C \in \mathbb{R}.$$

$$\int_0^{+\infty} \frac{x^2+5}{(x^3+15x+8)^{4/3}} \mathrm{d}x = \frac{1}{2}.$$

3. Let f be the function

$$f(x) = \frac{x^2-2x+5}{1-x}.$$

   (a) Determine its domain of definition, and find all its stationary points.
   (b) Determine the intervals where the function is increasing (respectively decreasing).
   (c) Determine the intervals of convexity and concavity of the function.

Solution (a) The function is well defined for all $x \in \mathbb{R} \setminus \{1\}$. The stationary points are $x = -1$ and $x = 3$.

(b) A sign table gives that
   i. For $x < -1$ and $x > 3$, $f'(x) < 0$ so the function is decreasing there;
   ii. For $-1 < x < 1$ and $1 < x < 3$, $f'(x) > 0$ so the function is increasing there.

(c) The function is convex in $(-\infty, 1)$ and concave in $(1, +\infty)$.

4. The expression

$$y^3 - x^2 y = 6,$$

defines y as a function of x: $y = y(x)$.

   (a) Find all the possible values of $y(1)$.

   (b) Find the Taylor polyomial of degree one of y about the point $x = 1$.

Solution (a) $y(1) = 2$.

(b) $p(x) = 2 + \frac{4}{11}(x-1)$.

5. Determine the minimum natural number $n \in \mathbb{N}$ such that the value of $\sum_{k=n}^{+\infty} 2^{-k}$ is smaller or equal than e^{-1000}.

   Tip: You may use that $(\ln 2)^{-1} \approx 1.44269504089$.

Solution $n = 1444$

6. Let

$$M = \begin{pmatrix} 2 & 1 \\ 4 & 6 \end{pmatrix}.$$

Find a matrix $N = (n_{i,j})$ with 2 rows och 3 colums such that $M \cdot N$ satisfies

$$M \cdot N = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Solution

$$N = \begin{pmatrix} \frac{3}{4} & -\frac{1}{8} & -1 \\ -\frac{1}{2} & \frac{1}{4} & 1 \end{pmatrix}.$$

7. Let $F(x,y) = x^2 + xy - 5x + 2y^2 - 6y$.

   (a) Find all stationary points for this function and determine whether they are local maximum, minimum, or saddle points. Does the fuction have a global maximum for $(x,y) \in \mathbb{R}^2$? Argument your answer.

   (b) Find the maximum and minimum value of F on the set

$$L = \{(x,y) : x \geq 0, y \geq 0, x + y = 5\}.$$

Solution There is only one stationary point $x = (2,1)$, and it is a global minimum point.

The function has no global maximum, since $\lim_{x \to +\infty} F(x,0) = +\infty$.

The maximum value of f on L is 20, and the minimum $-25/4$.

GOOD LUCK!
