

Instructions:

- In case of ambiguity, one has to refer to the ENGLISH version of this exam.
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The only approved calculator are those that will be handed by the personnel in the exam room. The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page, and you can use both sides of your paper)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

(1) (5pt) Compute the degree 3 Taylor polynomial of the function $f(x) = e^{-x^2+1}$, around the point $x_0 = 1$, and use it to give an approximation of $f(1.1)$.

(2) **Geometric sums**

(a) (1 pt) Compute

$$\sum_{k=4}^{90} (1.02)^k$$

(b) (2 pts) You are a financial adviser. Your client is thinking of investing \$600 at the end of every month (with the first payment right now) for the next 5 years with the invested funds earning 4.8% compounded monthly. Compute how much money the client has at the end of the 5 years.

(c) (2 pts) At the end of the five years, the same client tells you that they want to use the funding to make presents: they want to draw the same sum at the end of each year, with the first withdrawal in 1 year (so the capital accumulates 1 year of interest). How much can they take out if they want to make presents for the next 10 years?

(3) Consider the function $f(x) = xe^{-100x}$.

(a) (2 pts) Find where the function is increasing or decreasing. Find the critical points and determine their type.

(b) (1pt) Find in which intervals the function is concave or convex.

(c) (2 pts) Compute $\lim_{x \rightarrow \pm\infty} f(x)$ and sketch the graph of $f(x)$ (in order to get full points the graph must be consistent with your finding in (1) and (2). Critical points must be clearly visible.)

(4) Compute the following integrals:

(a) (3pt) $\int (e^{-t+1} + 6(2t+3)^5) dt,$

(b) (2pt) $\int_1^e \ln(x^2)xdx.$

(5) Consider the matrix

$$A = \begin{pmatrix} 1 & 3c & 0 \\ 0 & 1 & -3c \\ 1 & 0 & 1 \end{pmatrix}$$

(a) (2 pt) Compute the determinant of A , $|A|$ as a function of c .

(b) (1 pt) Find all the values of c for which A is not invertible.

(c) (2 pt) Determine whether the following linear system has 1, 0, or infinitely many solutions. In case there is just one solution, find this.

$$\begin{cases} x + 3y & = 2 \\ & y - 3z = 1 \\ x & + z = 4 \\ 2x + 4y - 2z & = 4 \end{cases}$$

(6) Consider the two variables function

$$f(x, y) = xy e^{-xy}$$

(a) (2pt) Find all the critical points of $f(x, y)$ and determine their type.

(b) (2pt) Consider now

$$D = \{(x, y) | 0 \leq x \leq 2, 0 \leq y \leq 2 - x\}.$$

Determine the maximum and minimum values taken by $f(x, y)$ on the *boundary* of the D .

(c) (1 pt) Determine the minimum and the maximum value of $f(x, y)$ on D .

GOOD LUCK!!!

Svenska Texten

- (1) (5pt) Beräkna det tredje gradens Taylors polynom till funktionen $f(x) = e^{-x^2+1}$, omkring punkten $x_0 = 1$. Använd det för att approximera $f(0.1)$.

(2) Geometrisk summor

- (a) (1 pt) Beräkna

$$\sum_{k=4}^{90} (1.02)^k$$

- (b) (2 pts) Du ska hjälpa en kund med sina investeringar. Kunden tänker investera \$600 i slutet av varje månad, varje månad under 5 år. Konton ger 4.8% årlig ränta, beräknad månadsvis (i slutet av månaden). Beräkna hur mycket kunden kommer att tjäna efter 5 år om den första betalningen är nu.
- (c) (2 pts) Efter de 5 åren, vil den samma kunden använder pengar för att göra presenter. Hen vill uttara samma summa i slutet av varje år med den första uttag om ett år (kapitalen har 1 år ränta när kunden gör den första uttagen). Beräkna hur mycket hen kan uttara om hen ska göra presenter under 10 år.

- (3) Betrakta funktionen $f(x) = xe^{-100x}$.

- (a) (2pts) Hitta var funktionen är växande eller avtagande. Hitta alla kritiska punkter och bestäm deras typ.
- (b) (1pt) Hitta var funktionen är konkav eller konvex.
- (c) (2 pts) Beräkna $\lim_{x \rightarrow \pm\infty} f(x)$ och rita grafen till $f(x)$.

- (4) Beräkna de följande integralerna

(a) (3pt) $\int (e^{-t+1} + 6(2t+3)^5) dt$,

(b) (3pt) $\int_1^e \ln(x^2) x dx$.

- (5) Beträkta matrisen

$$A = \begin{pmatrix} 1 & 3c & 0 \\ 0 & 1 & -3c \\ 1 & 0 & 1 \end{pmatrix}$$

- (a) (2 pt) Beräkna determinanten till A , $|A|$ som en funktion av c .
- (b) (1 pt) Hitta alla värden till c sådana att A inte är invertibara.
- (c) (2 pt) Bestäm hur många lösningar (1, 0 eller odliga många) har den följande system. Om det finns bara ett, räkna det.

$$\begin{cases} x + 3y & = & 2 \\ & y - 3z & = & 1 \\ x & & +z & = & 4 \\ 2x + 4y - 2z & = & 4 \end{cases}$$

- (6) Betrakta den följande funktionen i två variabler

$$f(x, y) = xye^{-xy}$$

- (a) (2pt) Hitta alla de kritiska punkter och bestäm deras typ.

(b) (2pt) La

$$D = \{(x, y) | 0 \leq x \leq 2, 0 \leq y \leq 2 - x\}.$$

Beräkna den största och den minsta värden till $f(x, y)$ på *gränsen* till D .

(c) (1 pt) Beräkna den största och minsta värden till $f(x, y)$ på D .

Lycka Till!!!