MM5021-ST23: Some tips for Reading the course-book

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The course-book is popularly know as "baby Rudin, and it is probably one of the most famous undergraduate mathematics books ever written, that has been around and used in Real Analysis courses, educating several generations of Mathematicians all over the world since 1953. But beware, it is not a novel, and although provides elegant and concise proofs to all the statements, it requires the reader to take a very pro-active role filling details.

One of the goals of this course is for the student to learn to read and understand the fundamental of Real Analysis. So, you are expected to read the coursebook carefully! This is no easy thing and this course is generally considered to be quite challenging. Reading mathematics substantially differs from other types of reading, as that requires more effort than just skimming through the text. Mathematics books are not novels, so their reading must often be slow and careful. Reading takes time!

We gather below some tips for helping you reading mathematics from the course-book, that may be applied to reading any mathematics book:

- **Focus on concepts, not exercises.** The most important material in math books is usually found in the prose, not in the exercises at the end of the section.

  The approach of reading a book differs from the one that many follow in lower level courses: one uses the book as a list of standard exercises, only skimming through the text trying to find similar exercises. Don't expect this approach to work any longer!

  You must pay more attention to the concepts and proofs, and eventually, attempt solving exercises. Mathematics is about ideas. If you can learn the key concepts, you will be able to solve any type of problem (including ones you have never seen before) that involves those concepts.

  Identifying which ones are the key concepts is part of the reading process.

- **Take your time to read the text and read it more than once.** Reading mathematics, is not like reading a novel. You need to go through the text several times, aiming each time to go deeper in the text.

Make sure to allocate regular times for reading the text, without distractions and/or pressure. Know yourself and pace your reading to your capabilities. Waiting for a last minute quick read is usually a bad strategy for understanding.

- **When reading through for the first time, scan for big ideas.** The first time you read through a chapter/section of the book, ask yourself: "What is the main point of the chapter?" Aim to form a big picture. The details are important, but you need to be aware of the forest first before focusing on the trees.

  If you get stuck, you should not stop immediately. Try to read on. Often the next sentences and/or examples can help you.

  Try to sort the problem yourself, but if even after going ahead in your reading you are still stuck, ask for assistance!

- **After the second time through, fill in details.** After you get the big picture, you should then look at the details. Take some time to think about each of the definitions, theorems, and formulas you encounter (more on this later).

- **Read with paper and pen.** As you are reading through the text, you should be writing your own set of lecture notes, completing details from the course-book and verifying any parts of which you are sceptical.

  Check any calculations. Rewrite definitions and theorems in your own words. Make pictures that illustrate the ideas (these pictures can help you understanding).

  See if you can come up with your own examples. Ask yourself about special cases of the theorems you read.

- **Be an active reader.** Reading Mathematics is not at all a linear experience. Understanding the text requires cross references, scanning, pausing and revisiting. Try to connect what you read with what you already know. You may ask yourself (some of) the following questions:

  1. Why is this idea true?
  2. Do I really believe it?
  3. Could I convince someone else that it is true?
  4. Why didn't the author use a different argument?
  5. Do I have a better argument or method of explaining the idea?
  6. Why didn't the author explain it the way that I understand it?
  7. Is my way wrong?
  8. Do I really get the idea?
  9. Am I missing some subtlety?
  10. Did this author miss a subtlety?
  11. If I can't understand the point, perhaps I can understand a similar but simpler idea? Which simpler idea?
  12. Is it really necessary to understand this idea?
  13. Can I accept this point without understanding the details of why it is true?
  14. Will my understanding of the whole story suffer from not understanding why the point is true?

When reading the statement of result, try to think ahead the text, and attempt to see how you could provide a proof? (This are usually the best exercises that you can attempt!)

- **Read the narrative.** There is a story to be told in mathematics. Try to follow the progression of ideas being told. Don't just skip to the formulas and examples, but instead follow the development of the ideas and concepts presented. Some motivating sentences may help you understanding an idea.

- **Study the examples.** What points do each of the examples illustrate? Some examples are extreme cases. Other examples are supposed to illustrate "typical" situations.

- **Read and make pictures.** There are not many illustrations in the course-book, but for those there, you should be asking yourself what features of the picture are important to the key concepts. Focus on how each picture illustrates a particular idea.

  Try to illustrate with a picture some of the ideas in the book on your own.

- **Learn the vocabulary and the language.** Pay attention to definitions and what they mean. Mathematics language is very precise, and a word in a mathematical context may have a different meaning than when it is used in everyday conversation. In mathematics, great care is taken to explicitly and precisely define the notions being considered. In addition, mathematical definitions and language are crafted in such a way to convey sophisticated notions in as simple and concise a manner as possible.

  The order in the language matters. Pay attention to the subtleties. If $P(\epsilon, \delta)$ is a statement depending on ϵ and δ, it is not the same to say that

  > "For all $\varepsilon > 0$, there exists $\delta > 0$ such that $P(\varepsilon, \delta)$ holds."

  than

  > "There exists $\delta > 0$ such that for all $\varepsilon > 0$, $P(\epsilon, \delta)$ holds."

- **Learn the theorems and what they mean.** Theorems are vital bricks to building mathematical knowledge. When you see a theorem in a mathematics text, look at it very closely. Ask yourself:

  1. What does it say?
  2. What are it's hypotheses?
  3. What implications does it have?
  4. Are there special cases you should be aware of?
  5. Can you think of examples to which the theorem applies?
  6. Can you think of examples that do not satisfy the hypotheses and the conclusion of the theorem?

- **Use the index and the appendices.** Know what every word means. Make sure that you understand all of the words and ideas. If there is a particular word which you do not know (or which you want to know better), look it up. Use the table of contents or the index to help you.

- **Make a note of things you do not understand, and ask for help afterwards.** Even after following all of the above advice, you might still find some of the ideas confusing. That is to be expected; material such as this is often hard to internalize when one first encounters it. If there is something that you do not understand, make a note of it. Write down any questions you may have. You then can bring up these issues with your classmates and instructors.

References

[1] Shai Simonson and Fernando Gouvea, *How to Read Mathematics*, `https://web.stonehill.edu/compsci/History_Math/math-read.htm`.

[2] Mark Tomforde, *Tips for reading your mathematics textbook*, `https://marktomforde.com/academic/undergraduates/MathReadingTips.pdf`.