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Direct Shrinkage Estimation of Large Dimensional Precision Matrix

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Abstract

In this work we construct an optimal shrinkage estimator for the precision matrix in high dimensions. We consider the general asymptotics when the number of variables $p \to \infty$ and the sample size $n \to \infty$ so that $p/n \to c \in (0, +\infty)$. The precision matrix is estimated directly, without inverting the corresponding estimator for the covariance matrix. The recent results from the random matrix theory allow us to find the asymptotic deterministic equivalents of the optimal shrinkage intensities and estimate them consistently. The resulting distribution-free estimator has almost surely the minimum Frobenius loss. Additionally, we prove that the Frobenius norms of the inverse and of the pseudo-inverse sample covariance matrices tend almost surely to deterministic quantities and estimate them consistently. At the end, a simulation is provided where the suggested estimator is compared with the estimators for the precision matrix proposed in the literature. The optimal shrinkage estimator shows significant improvement and robustness even for non-normally distributed data.

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