Singular Inverse Wishart Distribution with Application to Portfolio Theory

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Abstract

The inverse of the standard estimate of covariance matrix is frequently used in the portfolio theory to estimate the optimal portfolio weights. For this problem, the distribution of the linear transformation of the inverse is needed. We obtain this distribution in the case when the sample size is smaller than the dimension, the underlying covariance matrix is singular, and the vectors of returns are independent and normally distributed. For the result, the distribution of the inverse of covariance estimate is needed and it is derived and referred to as the singular inverse Wishart distribution. We use these results to provide an explicit stochastic representation of an estimate of the mean-variance portfolio weights as well as to derive its characteristic function and the moments of higher order.

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