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An epidemic in a dynamic population with importation of infectives

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Abstract

Consider a large uniformly mixing dynamic population, which has constant birth rate and exponentially distributed lifetimes, with mean population size n. A Markovian SIR (susceptible $\rightarrow$ infective $\rightarrow$ recovered) infectious disease, having importation of infectives, taking place in this population is analysed. The main situation treated is where $n \rightarrow \infty$, keeping the basic reproduction number R_0 as well as the importation rate of infectives fixed, but assuming that the quotient of the average infectious period and the average lifetime tends to 0 faster than $1/\log n$. It is shown that, as $n \rightarrow \infty$, the behaviour of the 3-dimensional process describing the evolution of the fraction of the population that are susceptible, infective and recovered, is encapsulated in a 1-dimensional regenerative process $S = \{S(t); t \geq 0\}$ describing the limiting fraction of the population that are susceptible. The process S grows deterministically, except at one random time point per regenerative cycle, where it jumps down by a size that is completely determined by the waiting time since the previous jump. Properties of the process S, including the jump size and stationary distributions, are determined.

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