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Using separate exposure for IBNYR and IBNER in the Chain Ladder method

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Abstract

Consider claims reserving based on incurred claim costs. For this case, Schnieper (1991) suggested a reserving method for separating the "true" unknown claims reserve (IBNYR) from the development of incurred claims (IBNER). While this is interesting in its own right, we suggest that the most important feature of the method is that it allows us to use a prior volume measure, such as premiums, as exposure for unknown claims, in contrast to the Chain ladder method that takes incurred claims as exposure for the combined IBNYR and IBNER reserve.

We present a new method using the same two data triangles as Schnieper's method, but with incurred claims as exposure for both unknown and known claims, separately. The resulting ultimate claim cost is identical to the one from the Chain ladder. Hence, the method provides a way to split the Chain ladder reserve into known and unknown claims. Together with Schnieper's method, we also get a framework for choosing the proper exposure measure for IBNYR, while always keeping incurred claims as exposure for IBNER.

KEY WORDS: Claims reserving, Case reserves, True IBNR, RBNS, Schnieper's method, Unknown claims.

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1 Introduction

In this paper we consider non-life claims reserving based on incurred claim costs (rather than paid claims). We follow Wütrich (2015) and use the acronym IBNYR (Incurred But Not Yet Reported) for the unknown claims and RBNS (Reported But Not Settled) for claims that are known but not completely paid. This divides the claim reserve into the IBNYR reserve and the RBNS reserve. The latter can be further divided into the aggregation of the individual case reserves and the IBNER (Incurred But Not Enough Reported) reserve which we take to mean the actuary's adjustment to the case reserves on a collective basis. So the object of the actuary is to estimate the IBNYR and the IBNER reserves. The sum of these is often referred to as the IBNR (Incurred But Not Reported) reserve, but this term is ambiguous since it is sometimes taken to mean the IBNYR reserve only.

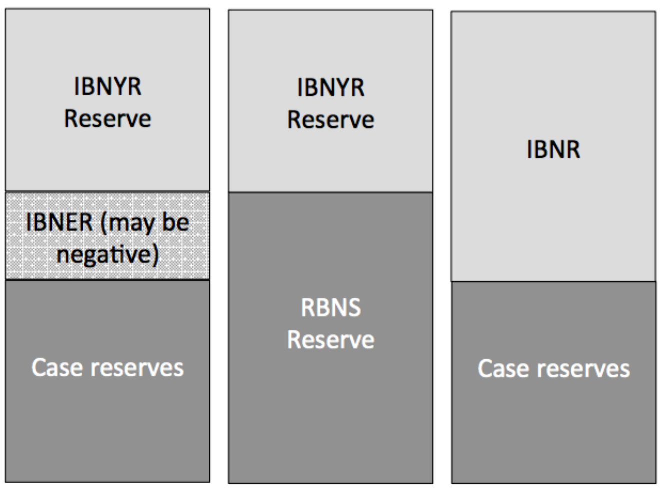

Figure 1.1: Different divisions of the claims reserve.

Schnieper (1991) suggested a reserving method that separates the IBNYR ("true IBNR") and IBNER reserves. In this paper we will argue that the most important feature of this method is that it allows the use of a prior volume measure, such as premiums, as exposure for IBNYR, while having incurred claim cost as the exposure for IBNER. This is in contrast to the Chain ladder method (CL), which in this context has incurred claim cost as exposure for the entire IBNR, i.e. for IBNER and IBNYR together. We will also argue that Schnieper's claim of separating IBNYR and IBNER is completely justified only under an additional assumption of incurred claims reported late having the same further development as those reported earlier. Whether this assumption holds true or not, Schnieper's method can always be used to change the exposure for IBNYR and this is potentially useful in applications.

We present a new method, using the same two data triangles as Schnieper's method, with incurred claims as exposure for IBNYR as well as IBNER. The resulting ultimate claim cost is identical to the one from the Chain ladder. Hence, the method can either be seen as providing a split of the Chain ladder IBNR reserve into IBNYR and IBNER, or as a way to change the exposure for IBNYR in Schnieper's method from a prior volume measure (say premiums) to the latest known incurred claim cost. The split into IBNYR and IBNER is strictly valid under the same assumption as Schniper's method.

This gives us a framework for claims reserving where we follow the development of reported claims in the usual chain-ladder way, while unknown claims are modelled as proportional to the premiums or any other prior exposure

(Schnieper's method) or the latest incurred claims (our variation of the chain ladder). The choice should depend on the application at hand.

The methods are illustrated by a numerical example from personal accident insurance. This is performed in a simple Excel sheet, illustrating how easy the method is to apply, once you have the two required data triangles.

Liu and Verrall (2009a) brought attention to the Schnieper model, noting that "it has not been considered any further since it was published". They presented estimates of its prediction error and discussed possible extensions of the method, in other directions than the present paper. Liu and Verrall (2009b) continued by giving the full predictive distribution, by bootstrapping the two separate triangles of the method. Liu and Verrall (2009a) notes that Schnieper's method "was specifically designed with reinsurance data in mind, but it is possible that it could be useful for other types of data as well." The truth of this conjecture is demonstrated by Flodström (2013, in Swedish) who presents an application to personal accident insurance.

Martínez-Miranda, Nielsen and Verrall (2012) presented a method called Double Chain Ladder (DCL) for splitting the Chain ladder estimate into IBNYR and RBNS, based on the paid and counts triangles. This method is not designed for altering the exposure, but as for the separation of IBNYR claims in the reserve, the DCL and our method may be seen as complimentary: the DCL is designed for paid claims and our method for incurred claims. Note that Martínez-Miranda et al. use assumptions that imply that the development of payments can not depend on how late the claim is reported; this is tantamount to the assumption for Schnieper's and our method mentioned above.

If a strict separation of IBNYR and IBNER/RBNS is sought, without special assumptions, it seems that one has to revert to three-dimensional reserving (the dimensions being accident, reporting and valuation period) as explained in Neuhaus (2004). However, this is at the cost of using quite complicated and potentially over-parameterised models, as indicated by Neuhaus.

Antonio and Plat (2014) considers micro-level reserving, i.e. reserving at the individual claim level. They use a Marked Poisson process approach, following a number of articles by Arjas, Haastrup and Norberg (for references, see Antonio and Plat). The simulation model used in that paper automatically gives a split of the reserve into IBNYR and RBNS, again at the cost of quite complicated models which are dependent on distributional assumptions.

In contrast, the method by Schnieper and the method presented here are as simple, non-parametric and transparent as a Chain Ladder run twice, once you have the required data.

2 The Chain ladder method

In this section we briefly recapitulate the standard Chain Ladder (CL) method. In the applications we have in mind, the data required is given in form of an incurred claims triangle. So we let C_{ij} denote the cumulated incurred claims (paid claims plus case reserves) in accident year i to the amount known after development year j. Here $i = 1, 2, \ldots, I$; and $j = 0, 1, \ldots, J-1$; for some I and J. For simplicity, we consider only the case is $I = J$, though that is not a necessary assumption.

Accident year	*Development year* 0	1	2	$\cdots$	$I-2$	$I-1$
1	C_{10}	C_{11}	C_{12}	$\cdots$	$C_{1,I-2}$	$C_{1,I-1}$
2	C_{20}	C_{21}	C_{22}	$\cdots$	$C_{2,I-2}$	
3	C_{30}	C_{31}	C_{32}	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$I-1$	$C_{I-1,0}$	$C_{I-1,1}$				
I	$C_{I,0}$					

Table 2.1: Incurred claims triangle.

The calendar years are on the diagonal. The "CL idea", in the wording of Wütrich (2015), is that all accident years behave similarly and that we have approximately

$$C_{i,j} \approx C_{i,j-1}\, f_j$$

This very old idea was turned into the famous "basic chain ladder assumption" by Mack (1993), which in our notation can be written

(C1) For some parameters $f_j > 0$, and all $i = 1, \ldots, I$ and $j = 1, \ldots, I-1$, we have:

$$E[C_{ij} | C_{i0}, \ldots, C_{i,j-1}] = C_{i,j-1}\, f_j$$

Mack also assumes the following:

(C2) The random variables for the accident years are independent, i.e. the random vectors $\{C_{i0}, \ldots, C_{i,I-1}\}$ are independent of each other.

The Chain ladder estimator is

$$\hat{f}_j = \frac{\sum_{i=1}^{I-j} C_{i,j}}{\sum_{i=1}^{I-j} C_{i,j-1}} \qquad j = 1, \ldots, I-1 \tag{2.1}$$

Mack (1993) showed that, under (C1) and (C2), the estimators $\hat{f}_j$ are unbiased and uncorrelated for different j.

The value in row i on the last observed diagonal, where $i + j = I$, is $C_{i,I-i}$. By multiplying it by the successive development factors, we get an estimate of the ultimate claim cost $C_{i,I-1}$.

$$\hat{C}_{i,I-1} = C_{i,I-i}\, \hat{f}_{I-i+1} \cdots \hat{f}_{I-1} \qquad i = 2, \ldots, I-1 \tag{2.2}$$

As noted by Mack (1993), it follows that $\hat{C}_{i,I}$ is an unbiased estimator (predictor) of the ultimate claim cost $C_{i,I}$; in the meaning that they both have the same expectation, conditional on the data observed so far.

3 Schnieper's method

To introduce the method by Schnieper (1991), we need to split the cumulative incurred claims triangle in Table 2.1 in two: the new claims triangle and the run-off triangle. Let N_{ij} be the incurred claim cost for claims incurred in accident year i and reported during development year j, as recorded by the end of that year. Note that this is not the most recently known incurred cost for these claims, but the status by the end of the year when the claims were reported. If our claims data base contains information about the reporting date for each claim, and the changes in case reserves from year to year, we can compute the new claims triangle, the "N triangle" in Table 3.1.

Accident year	*Development year* 0	1	2	$\cdots$	$I-2$	$I-1$
1	N_{10}	N_{11}	N_{12}	$\cdots$	$N_{1,I-2}$	$N_{1,I-1}$
2	N_{20}	N_{21}	N_{22}	$\cdots$	$N_{2,I-2}$	
3	N_{30}	N_{31}	N_{32}	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$I-1$	$N_{I-1,0}$	$N_{I-1,1}$				
I	$N_{I,0}$					

Table 3.1: New claims triangle.

Note that $C_{i0} = N_{i0}$, for all i. When we go from development year $j-1$ to year j, the new claims N_{ij} are added to $C_{i,j-1}$, but there is also a change in the incurred claim cost for existing claims, which we denote by D_{ij}. Hence $C_{ij} = C_{i,j-1} + D_{ij} + N_{ij}$, by which we can compute D_{ij} indirectly through

$$D_{ij} = C_{ij} - C_{i,j-1} - N_{ij}\,; \quad i = 1, 2. \ldots, I; \quad j = 1, 2, \ldots, I-1 \qquad (3.1)$$

For $j = 0$ there is of course no changes to record; when needed we define $D_{i,0} \equiv 0$.

We are now ready to set up the pure development or run-off "D triangle" in Table 3.2. This is the triangle of changes to the incurred claim cost during a year, for claims already known at the beginning of that year. Note that we have chosen a somewhat different notation from that of Schnieper (1991).

We define $\mathcal{D}_k$ as all variables of interest known up to calendar year k. Here

Accident	*Development year*					
year	0	1	2	$\cdots$	$I-2$	$I-1$
1	D_{10}	D_{11}	D_{12}	$\cdots$	$D_{1,I-2}$	$D_{1,I-1}$
2	D_{20}	D_{21}	D_{22}	$\cdots$	$D_{2,I-2}$	
3	D_{30}	D_{31}	D_{32}	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$I-1$	$D_{I-1,0}$	$D_{I-1,1}$				
I	$D_{I,0}$					

Table 3.2: Run-off triangle.

it can be written as the collection of any two of our three triangles, e.g.,

$$\mathcal{D}_k = \{N_{ij}, D_{ij}; i + j \leq k\}$$

Schnieper's method also makes use of an exposure measure for IBNYR, which for accident year i is denoted E_i, with $E_i > 0$. Typical choices would be earned premium, sum insured or duration (number of insurance years) for the entire portfolio year i. We are now in the position of stating Schnieper's assumptions, in slightly different notation.

(A1) For some parameters $\lambda_j \geq 0$,

$$E[N_{ij}|\mathcal{D}_{i+j-1}] = E_i \lambda_j\,; \quad i = 1, 2.\ldots, I; \quad j = 1, 2.\ldots, I-1$$

(A2) For some parameters δ_j,

$$E[C_{i,j-1} + D_{ij}|\mathcal{D}_{i+j-1}] = C_{i,j-1}\,\delta_j;\,; \quad i = 1, 2.\ldots, I; \quad j = 1, 2, \ldots, I-1$$

(A3) The random variables for the accident years are independent, i.e. the random vectors $\{N_{i0}, D_{i0}, \ldots, N_{i,I-1}, D_{i,I-1}\}$ are independent of each other.

The similarity to Mack's (C1) is no coincidence: Mack (1993) explicitly credits Schnieper (1991) for making the "decisive step" towards his work. Note that we have written Schnieper's assumptions in a form that is more similar to Mack's than the original.

Note also that Schnieper inlcudes the case $j = 0$ in (A1), for use in pricing. Now this is a bit problematic, since, e.g., the premium volume is not fully known before the start of the insurance period. In our view, the exposure in (A1) should be known (non-random) by the end of the accident year for the use in our reserving from that point and on. For this reason, we prefer to make $j = 0$ an optional assumption (A0), and postpone it to Section 5.

It follows from (A1) and (A2) that

$$E[C_{i,j}|\mathcal{D}_{i+j-1}] = E[C_{i,j-1} + D_{i,j} + N_{i,j}|\mathcal{D}_{i+j-1}] = C_{i,j-1}\,\delta_j + E_i\lambda_j \tag{3.2}$$

This is in contrast to the Chain ladder where, in the current notation,

$$E[C_{i,j}|\mathcal{D}_{i+j-1}] = E[C_{i,j-1} + N_{i,j} + D_{i,j}|\mathcal{D}_{i+j-1}] = C_{i,j-1}\,f_j$$

Note that (A1) means that the expected cost for new claims is not influenced by the observed claims so far, but instead proportional to the earned premium, while in the Chain ladder the expected value of both N and D are dependent on $C_{i,j-1}$. We will investigate this in more detail in Section 4.

From assumptions (A1) and (A2) it is immediate that the following estimators are (conditionally) unbiased, given $\mathcal{D}_{i+j-1}$.

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{I-j} N_{ij}}{\sum_{i=1}^{I-j} E_i} \qquad j = 1, \ldots, I-1 \tag{3.3}$$

$$\hat{\delta}_j = \frac{\sum_{i=1}^{I-j}(C_{i,j-1} + D_{ij})}{\sum_{i=1}^{I-j} C_{i,j-1}} = 1 + \frac{\sum_{i=1}^{I-j} D_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}} \qquad j = 1, \ldots, I-1 \tag{3.4}$$

While accident year $i = 1$ is assumed to be fully developed, for year $i = 2$ we can use (3.2) to find that

$$E[C_{2,I-1}|\mathcal{D}_I] = C_{2,I-2}\,\delta_{I-1} + E_2\lambda_{I-1}$$

An estimator $\hat{C}_{2,I-1}$ of the ultimate claim cost is found by plugging in the estimates $\hat{\lambda}_{I-1}$ and $\hat{\delta}_{I-1}$.

By using equation (3.2) and iterated expectation

$$\begin{aligned} E[C_{i,j+1}|\mathcal{D}_{i+j-1}] &= E[E[C_{i,j+1}|\mathcal{D}_{i+j}]|\mathcal{D}_{i+j-1}] \\ &= E[C_{ij}\,\delta_{j+1} + E_i\lambda_{j+1}|\mathcal{D}_{i+j-2}] \\ &= C_{i,j-1}\,\delta_j\delta_{j+1} + E_i[\lambda_j\delta_{j+1} + \lambda_{j+1}] \end{aligned}$$

This procedure can be used repeatedly to find an expression for the conditional expectation of the ultimate claim cost $C_{i,I-1}$ for the accident years $i > 2$,

$$\begin{aligned} E[C_{i,I-1}|\mathcal{D}_I] &= C_{i,I-i}\,\delta_{I-i+1}\cdots\delta_{I-1} \\ &+ E_i(\lambda_{I-i+1}\,\delta_{I-i+2}\cdots\delta_{I-1} \\ &+ \lambda_{I-i+2}\,\delta_{I-i+3}\cdots\delta_{I-1} \\ &\vdots \\ &+ \lambda_{I-2}\,\delta_{I-1} \\ &+ \lambda_{I-1})\,; \qquad i = 3, \ldots, I-1 \end{aligned} \tag{3.5}$$

Again we plugg in the estimates of λ and δ in order to get an estimator of the total reserve. Schnieper does not prove unbiasedness of this estimator, but a proof could be made along the following lines. Let $\hat{C}_{i,j}$ denote the estimator (predictor) of $C_{i,j}$ that is found by plugging in the parameter estimates in the right-hand side of (3.2). Liu and Verrall (2009a) notes that unbiasedness follows from the inductive property of the estimators

$$\hat{C}_{i,j+1} = \hat{C}_{i,j}\,\hat{\delta}_{j+1} + E_i\hat{\lambda}_{j+1} \tag{3.6}$$

Here $\hat{C}_{i,j}$ should be read as $C_{i,j}$ when we reach the first observed value on row i, which would be the starting point of a proof by induction.

Schnieper (1991) claims that the expression starting with E_i in (3.5) gives the IBNYR reserve and that the first term represents (the ultimate cost for) the known claims. If so, the IBNER adjustment to the incurred claims reserve is found by subtracting $C_{i,I-i}$, with the result is $C_{i,I-i}\,(\delta_{I-i+1}\cdots\delta_{I-1}-1)$. However, Schnieper's claim is only true under the following additional assumption, which we state in words, for the sake of not having to introduce tedious notation for this sole purpose.

(A4) The incurred claims of accident year i that are reported in development year j, N_{ij}, have the same expected further development from $j+1$ and onward as the incurred claims reported earlier $C_{i,j-1}$ have.

We will argue in Section 6 that this is a reasonable assumption for the application to personal accident insurance there. In other lines of business, such as property, it might be less realistic to assume (A4), since complicated claims can be expected to be reported more rapidly than simple ones, not so costly claims. However, even if (A4) is not true, (3.5) gives an unbiased estimator of the claim cost, since unbiasedness only depends on (A1) and (A2). The interpretation is that δ_j represents the average development of all claims reported by $j-1$ together, irrespective of the year of reporting, but only if (A4) holds true it also represents the separate development of new claims and older claims, respectively.

The practical implication of this is that we can safely use (3.5) to estimate the reserve and the IBNR, but in some cases the split of the IBNR reserve into IBNER and IBNYR will not be perfect.

4 The new method

We believe that the most important property of Schnieper's method is that it allows us to assume the unknown claim cost to be proportional to any non-random exposure, with the premium as the typical case. We will now present a method that allows us to use the random variable *incurred claims* as exposure in a similar way to Schnieper's use of E_i. It turns out that even if the model just plugs in $C_{i,j-1}$ instead of E_i in (A1), the resulting estimator can not be found by the same simple plug in.

So we keep the assumptions (A2) and (A3) from Schnieper's method, but we change (A1) into the following assumption.

(A1*) For some parameters $\lambda_j > 0$,

$$E[N_{ij}|\mathcal{D}_{i+j-1}] = C_{i,j-1}\lambda_j\,; \quad i = 1, 2 \ldots, I; \quad j = 1, 2 \ldots, I-1$$

It follows that

$$E[C_{i,j}|\mathcal{D}_{i+j-1}] = E[C_{i,j-1} + D_{i,j} + N_{i,j}|\mathcal{D}_{i+j-1}] = C_{i,j-1}\,(\delta_j + \lambda_j) \qquad (4.1)$$

so that assumption (C1) for the Chain ladder is fulfilled with $f_j = \delta_j + \lambda_j$. Furthermore, (C2) follows directly from (A3). So under (A1*), (A2) and (A3), Mack's assumptions for the Chain ladder are fulfilled.

In analogy to the estimators of Schnieper (1991), we suggests the following two estimators. It follows directly from assumptions (A1*) and (A2) that they are conditionally unbiased, given $\mathcal{D}_{i+j-1}$.

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{I-j} N_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}} \qquad j = 1, \ldots, I-1 \qquad (4.2)$$

$$\hat{\delta}_j = 1 + \frac{\sum_{i=1}^{I-j} D_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}} \qquad j = 1, \ldots, I-1 \qquad (4.3)$$

(The δ estimates are of course identical to those for Schnieper's method.) Note that

$$\begin{aligned}\hat{\lambda}_j + \hat{\delta}_j &= \frac{\sum_{i=1}^{I-j} N_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}} + \frac{\sum_{i=1}^{I-j}(C_{i,j-1} + D_{ij})}{\sum_{i=1}^{I-j} C_{i,j-1}} = \frac{\sum_{i=1}^{I-j}(C_{i,j-1} + D_{ij} + N_{ij})}{\sum_{i=1}^{I-j} C_{i,j-1}} \\ &= \frac{\sum_{i=1}^{I-j} C_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}} = \hat{f}_j\,; \qquad j = 1, \ldots, I-1\,. \qquad (4.4)\end{aligned}$$

i.e., the sum of our estimators is the original CL estimator, consistent with the relation $f_j = \delta_j + \lambda_j$.

By using iterated expectation and equation (4.1)

$$\begin{aligned} E[C_{i,j+1}|\mathcal{D}_{i+j-1}] &= E[E[C_{i,j+1}|\mathcal{D}_{i+j}]|\mathcal{D}_{i+j-1}] = E[C_{ij}(\delta_{j+1}+\lambda_{j+1})|\mathcal{D}_{i+j-1}] \\ &= C_{i,j-1}(\delta_j+\lambda_j)(\delta_{j+1}+\lambda_{j+1}) \\ &= C_{i,j-1}\,\delta_j\delta_{j+1} + C_{i,j-1}\lambda_j\delta_{j+1} + C_{i,j-1}(\delta_j+\lambda_j)\lambda_{j+1} \end{aligned}$$

where the first term corresponds to the incurred claims after two more years of development; the second term represents the new claims the next year, developed one year; and the third term is the new claims in the second year.

Together with (4.1) for $i = 2$, this can be used for $I > 2$ as in Schnieper's method to find

$$\begin{aligned} E[C_{i,I-1}|\mathcal{D}_I] &= C_{i,I-i}(\delta_{I-i+1}+\lambda_{I-i+1})(\delta_{I-i+2}+\lambda_{I-i+2})\cdots(\delta_{I-1}+\lambda_{I-1}) \\ &= C_{i,I-i}f_{I-i+1}f_{I-i+2}\cdots f_{I-1}\,; \qquad i=2,\ldots,I-1 \end{aligned} \tag{4.5}$$

By plugging in our estimates of λ and δ we get an estimator of the ultimate claim cost that is identical to that for the Chain ladder in (2.2). However, under assumption (A4), we can use the new method to separate the IBNER and IBNYR reserves. The RBNS claims reserve is then found by taking out the first term in an expansion of the middle member of (4.5), and plugging in the estimated parameters, i.e.

$$C_{i,I-i}\hat{\delta}_{I-i+1}\hat{\delta}_{I-i+2}\cdots\hat{\delta}_{I-1} \tag{4.6}$$

and the IBNER reserv is this minus the case reserves. The unknown claim cost IBNYR is then the CL estimate of the ultimate minus (4.6).

The discussion on (A4) at the end of Section 3 applies equally well here. Since the overall estimate is the ordinary CL estimate, our method gives a possibility to split the CL estimate into IBNYR and IBNER, if (A4) applies (approximately). Note that if the case estimates are unbiased, (A4) is trivially fulfilled. In any case, the separation of f_j into δ_j and λ_j may help in the analysis and improve the CL estimate when it comes to smoothing, tail estimation or expert judgement.

Another conclusion is that the difference in reserve estimates between CL and Schnieper's method is entirely due to the change of exposure for IBNYR and nothing else. The choice between the two is a choice of whether the actuary wants to assume that new claims are proportional to incurred claims or to the premium. A wish to separate IBNYR can be met with any of these choices, as explained above.

4.1 Individual factors

Note that, as with the chain ladder, the Schnieper estimators can be rewritten as weighted sums of individual unbiased estimators. Similarly, the estimators in (4.2) and (4.3) may trivially be written

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{I-j} C_{i,j-1}\,\hat{\lambda}_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}}\,; \qquad \hat{\lambda}_{ij} = \frac{N_{ij}}{C_{i,j-1}} \tag{4.7}$$

$$\hat{\delta}_j = \frac{\sum_{i=1}^{I-j} C_{i,j-1}\,\hat{\delta}_{ij}}{\sum_{i=1}^{I-j} C_{i,j-1}}\,; \qquad \hat{\delta}_{ij} = 1 + \frac{D_{ij}}{C_{i,j-1}} \tag{4.8}$$

From a practical point of view, the individual estimators $\hat{\lambda}_{ij}$ and $\hat{\delta}_{ij}$ are interesting as tools of analysis. For example, one may search for trends over

i, for outliers or shifts. Since each individual estimator is unbiased, we may form alternative estimators by only including some of them in the estimator, typically the latest ones.

5 Loss ratio estimation

The reader who is only interested in reserving may skip this section without loss.

If we are considering estimating the loss ratio (LR) before the insurance year starts, consistent with the reserving, we can add the following assumption (A0) to (A1)-(A3).

(A0) With a parameter $\lambda_0 \geq 0$,

$$E[N_{i0}] = E_i \lambda_0 \, ; \quad i = 1, 2. \dots, I;$$

Note that in Schnieper (1991) this assumption is part of (A1). The parameter λ_0 can be estimated by adding the case $j = 0$ to (3.3). The expected claim cost is then, in the Schnieper case with E_i taken as the premium

$$\begin{aligned} E[C_{i,I-1}] &= E_i(\lambda_0 \, \delta_1 \cdots \delta_{I-1} \\ &+ \lambda_1 \delta_2 \cdots \delta_{I-1} \\ &+ \lambda_2 \delta_3 \cdots \delta_{I-1} \\ &\vdots \\ &+ \lambda_{I-12} \delta_{I-1} \\ &+ \lambda_{I-1}) \end{aligned} \tag{5.1}$$

and the expected LR is found by dividing this by E_i. In our method, (A1*)-(A3), there is no natural extension of (A1*) to the case $j = 0$, but we may of course assume in that case too that the claim cost is proportional to the premium, i.e. that (A0) is in force.

Similarly to the derivation of (4.5) we find the following expression, using (A0) and the fact that $C_{i0} = N_{i0}$,

$$\begin{aligned} E[C_{i,I-1}] &= E_i\lambda_0(\delta_1+\lambda_1)(\delta_2+\lambda_2)\cdots(\delta_{I-1}+\lambda_{I-1}) \qquad (5.2) \\ &= E_i\lambda_0 f_1 f_2\cdots f_{I-1} \end{aligned}$$

so that the LR is $\lambda_0 f_1 f_2 \cdots f_{I-1}$. Of course, here $E_i\lambda_0$ could be replaced by any estimator of the claim cost as it is *after the first years development only*, e.g. one using a compound Poisson distribution, otherwise more commonly used for the ultimate cost. Cf. the discussion of the one-year perspective in contrast to the ultimate perspective in Ohlsson & Lauzeningks (2009), in another context.

6 Application to personal accident insurance

We consider personal accident insurance that gives income protection in the form av a limited lump-sum. For some such products there is a substantial amount of late reported claims, giving rise to IBNYR even after several years of development. The main reason is that childhood accidents may cause disability later in life; claims can then be made with reference to medical records from the accident year. Since the claim handlers have similar information in terms of medical records and employment history for these late reported claims and the ones that are reported earlier but still open, we expect assumption (A4) to hold true in this case.

We will give a simple example that illustrates the use of the methods in the present paper, based on data from Länsförsäkringar Alliance in Sweden. For confidentiality reasons we will not disclose the exact product(s) involved nor the name of the insurance company within the alliance from which the data emanates. Furthermore, the data has been reduced to ten development years and all amounts have been transformed to an undisclosed currency unit.

The three triangles of claims data (C, N and D) can be found at the end of the paper.

We apply three methods: the ordinary Chain ladder on the C triangle, our method with separation of IBNYR and IBNER in the Chain ladder, introduced in Section 4, and finally the method by Schnieper (1991) as described in Section 3, using premium volume as exposure for IBNYR. The resulting development factors, computed by (2.1), (3.3), (3.4) and (4.2), are given in Table 6.1.

Table 6.1: Development factors for the three methods

Dev. Year	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
f **for CL**	1,514	1,184	1,102	1,037	1,013	1,011	1,009	0,983	1,006
λ **our meth.**	0,374	0,118	0,073	0,049	0,035	0,026	0,040	0,026	0,015
λ **Schniep.**	0,130	0,062	0,046	0,034	0,026	0,020	0,031	0,019	0,012
δ **for both**	1,139	1,066	1,028	0,988	0,978	0,985	0,969	0,958	0,990
Our $\lambda + \delta$	1,514	1,184	1,102	1,037	1,013	1,011	1,009	0,983	1,006

In Table 6.2 we present the result of splitting the Chain ladder result into

IBNER and IBNYR, expecting (A4) to be valid in this case, as argued at the beginning of this section.

Table 6.2: Splitting the Chain ladder result into IBNER and IBNYR

Accident year	*Latest incurred*	*Chain ladder* *IBNR*	*Ultimo*	*Our method* *IBNER*	*IBNYR*	*Ultimo*
2005	25 160	0	25 160	0	0	25 160
2006	22 871	130	23 001	-221	351	23 001
2007	31 128	-347	30 781	-1 606	1 259	30 781
2008	23 548	-54	23 494	-1 907	1 853	23 494
2009	26 848	239	27 087	-2 550	2 789	27 087
2010	25 423	553	25 976	-2 928	3 481	25 976
2011	24 838	1 484	26 321	-3 129	4 613	26 321
2012	26 264	4 396	30 660	-2 663	7 059	30 660
2013	21 653	8 273	29 926	-912	9 185	29 926
2014	13 168	14 379	27 547	1 202	13 177	27 547
Sum	*240 901*	*29 052*	*269 953*	*-14 715*	*43 767*	*269 953*

The ultimate claim cost is of course the same for both methods. The IBNER is seen to be mostly negative – probably an effect of the claims handlers trying to be on the safe side in cases of doubt. It should be noted that the year 2005 may not be fully run-off in practice, since we have omitted data beyond the tenth development year. Note that the CL seems to indicate that there might just be random variation in the earliest years, with factor 7 being above 1, factor 8 being below 1 and then factor 9 being larger than 1 again. However, the split shows that there is still a substantial amount of IBNYR for those years, with IBNER going in the opposite direction and presumably stabilising by factor 9 which is at 0,99. Hence there might be a need for a tail in λ here, i.e. a development beyond the ten years of the triangle. Note that even if a split into IBNER and IBNYR is not necessary here, since we get the same total result as in the CL, the possible need of having a tail is important in assessing the final best estimate. The result of the split may also be a reason to look into the case reserves and try to make them more unbiased.

We move on to Table 6.3 with a comparison to Schnieper's method.

Table 6.3: Comparison of our method to Schnieper's method

Accident	*Our method*			*Schnieper's method*		
year	*IBNER*	*IBNYR*	*Ultimo*	*IBNER*	*IBNYR*	*Ultimo*
2005	0	0	25 160	0	0	25 160
2006	-221	351	23 001	-221	406	23 055
2007	-1 606	1 259	30 781	-1 606	1 109	30 632
2008	-1 907	1 853	23 494	-1 907	2 160	23 801
2009	-2 550	2 789	27 087	-2 550	2 865	27 163
2010	-2 928	3 481	25 976	-2 928	3 866	26 360
2011	-3 129	4 613	26 321	-3 129	5 129	26 837
2012	-2 663	7 059	30 660	-2 663	7 025	30 626
2013	-912	9 185	29 926	-912	9 632	30 373
2014	1 202	13 177	27 547	1 202	16 106	30 477
Sum	*-14 715*	*43 767*	*269 953*	*-14 715*	*48 297*	*274 484*

Here, the IBNER is the same and the difference is that the Chain ladder assumes the IBNYR to be proportional to the latest incurred, while Schnieper's method assumes that it is proportional to the premium volume. The resulting IBNYR are different, but not to a large extent, except for the latest year 2014. Because of the low amount incurred after the first year, it is probable that Schnieper gives the more reliable estimate here.

An Excel sheet where the above calculations are carried out can obtained from the author upon request.

Flodström (2013) compares the ordinary Chain ladder to Schnieper's method for two personal accident products. In this full-scale application the estimators include the latest seven individual factors (cf. Section 4.1) and are smoothed by cubic splines. For the first product, the difference between the results is small, only 0,92 % of the ultimate cost, which is 4,86 % of the reserve in this case.

For the second product, though, the difference is substantial with ultimate costs that differ by 7,4 % of the ultimate cost or 13,84 % of the reserve. Furthermore, Schnieper's method indicates the need for a tail in the λ's while that need was blurred by the addition of positive IBNYR and negative IBNER in the ordinary CL method. With a tail in the λ's the difference is 12,73 % of the ultimate cost and a large 24,73 % of the reserve.

This shows that there can be a substantial difference between choosing premiums or incurred claim costs as exposure for IBNYR claims and that the analysis of IBNYR separate from IBNER may give insights that has an important impact on the final estimates. By repeating the method over a sequence of calendar years, Flodström gets some evidence that Schnieper should be the more reliable estimator here.

7 Discussion and conclusions

Often incurred claim cost contains more information than paid claims, and is then preferred by the reserving actuary. Instead of just using a chain ladder estimate in this case, we suggest a split in IBNER and IBNYR along the lines of Schnieper (1991). The actuary then has the choice of using premiums or some other known volume measure as exposure for IBNYR (Schnieper's method) or using the latest incurred claim cost as that exposure (our method).

The split into IBNER and IBNYR reserves may give valuable insight into the development process that may be important for the final estimates, as discussed in Section 6. We have seen that we only get a strict split into IBNER and IBNYR under the extra assumption (A4). While this might be (approximately) fulfilled in many cases, the total reserve remains unbiased even if it is not. The choice of exposure is still relevant, too.

In practice, the actuary is often asked to provide "IBNR" (here meaning IBNYR) while it is understood that the case reserves give the RBNS reserve. By using the split into IBNER and IBNYR, the actuary can both fulfill this request and argue for an IBNER adjustment of the case reserves whenever necessary, based on examination of the δ's.

Schnieper's method and our variation thereof have been discussed in terms of observed incurred claims. Mathematically, incurred claims could be substituted by paid claims above, without altering the equations. However, we believe that this has limited applications, unless large part of the ultimate claim is paid the first year. In particular, it seems less likely that (A4) will be fulfilled for paid claims.

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Table 7.1: Personal accident: Incurred claims (the C triangle)

Accident	*Development year*									
year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2005	12 893	18 064	20 263	22 194	23 768	24 148	24 654	24 779	25 018	25 160
2006	12 248	19 539	22 900	24 922	25 440	24 617	24 407	23 925	22 871	
2007	13 189	20 580	24 497	27 737	28 582	30 165	30 064	31 128		
2008	10 856	17 049	20 831	22 252	23 394	22 602	23 548			
2009	13 132	19 227	22 635	25 232	25 580	26 848				
2010	12 169	18 509	21 894	24 397	25 423					
2011	13 432	18 696	22 732	24 838						
2012	13 808	22 073	26 264							
2013	14 151	21 653								
2014	13 168									

Table 7.2: Personal accident: New claims triangle (the N triangle) and premiums

Accident	*Development year*										*Premium*
year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*(exposure)*
2005	12 893	4 281	1 970	1 480	1 543	741	1 005	895	870	384	31 804
2006	12 248	5 836	2 502	2 065	1 316	479	702	1 042	376		33 604
2007	13 189	5 327	2 850	2 248	1 549	1 942	304	1 224			35 838
2008	10 856	4 421	2 536	1 071	1 112	389	674				35 664
2009	13 132	4 308	1 987	1 972	982	893					36 460
2010	12 169	4 321	1 813	1 344	741						37 976
2011	13 432	4 275	2 532	1 258							38 820
2012	13 808	6 033	1 945								40 823
2013	14 151	4 577									42 229
2014	13 168										45 655

Table 7.3: Personal accident: Run-off triangle (the D triangle)

Accident	*Development year*									
year	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2005	0	891	228	452	31	-361	-499	-771	-631	-242
2006	0	1 455	859	-43	-798	-1 301	-913	-1 524	-1 430	
2007	0	2 065	1 066	992	-704	-359	-406	-160		
2008	0	1 773	1 245	350	30	-1 181	272			
2009	0	1 787	1 422	624	-633	375				
2010	0	2 020	1 571	1 160	284					
2011	0	989	1 503	847						
2012	0	2 232	2 247							
2013	0	2 925								
2014	0									