



# Exact and Asymptotic Tests on a Factor Model in Low and Large Dimensions with Applications

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## Abstract

In the paper, we suggest three tests on the validity of a factor model which can be applied for both small dimensional and large dimensional data. Both the exact and asymptotic distributions of the resulting test statistics are derived under classical and high-dimensional asymptotic regimes. It is shown that the critical values of the proposed tests can be calibrated empirically by generating a sample from the inverse Wishart distribution with identity parameter matrix. The powers of the suggested tests are investigated by means of simulations. The results of the simulation study are consistent with the theoretical findings and provide general recommendations about the application of each of the three tests. Finally, the theoretical results are applied to two real data sets, which consist of returns on stocks from the DAX index and on stocks from the S&P 500 index. Our empirical results do not support the hypothesis that all linear dependencies between the returns can be entirely captured by the factors considered in the paper.

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