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A Dynamic Erdős-Rényi Graph Model

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Abstract

In this article we introduce a dynamic Erdős-Rényi graph model, in which, independently for each vertex pair, edges appear and disappear according to a Markov on-off process.

In studying the dynamic graph we present two main results. The first being on how long it takes for the graph to reach stationarity. We give an explicit expression for this time, as well as proving that this is the fastest time to reach stationarity among all strong stationary times.

The second result concerns the time it takes for the dynamic graph to reach a certain number of edges. We give an explicit expression for the expected value of such a time, as well as study its asymptotic behavior. This time is related to the first time the dynamic Erdős-Rényi graph contains a cluster exceeding a certain size.

1 Introduction

The Erdős-Rényi graph, in this text called the *static* Erdős-Rényi graph, is a well-studied model for random graphs, which is either (i) consisting of n vertices and k edges, where the edges are assigned uniformly to the $\binom{n}{2}$ vertex pairs—this graph model is denoted $G(n, m)$; or (ii) consisting of n vertices where edges are assigned independently between vertex pairs with probability p—this graph model is denoted $G(n, p)$, see [2] for more details and many properties of the model. In this article we introduce a natural dynamic version of such a model: the dynamic Erdős-Rényi graph.

Before moving on we set some notation: Throughout $N = \binom{n}{2}$. Furthermore. we use the asymptotic order notation: $f(n) = O(g(n))$ if and only if $|f(n)| \leq M|g(n)|$ for large n and some $M < \infty$; $f(n) = \Theta(g(n))$ if and only if both $f(n) = O(g(n))$ and $g(n) = O(f(n))$; finally, $f(n) = o(g(n))$ if and only if $\lim_{n\to\infty} \frac{|f(n)|}{|g(n)|} = 0$. Throughout, all asymptotic's are for the limit $n \to \infty$.